NAG Toolbox

nag_matop_complex_gen_rq (f01rj)

1 Purpose

nag_matop_complex_gen_rq (f01rj) finds the RQ factorization of the complex m by n ($m \leq n$), matrix A, so that A is reduced to upper triangular form by means of unitary transformations from the right.

2 Syntax

```
[a, theta, ifail] = nag_matop_complex_gen_rq(a, 'm', m, 'n', n)
[a, theta, ifail] = f01rj(a, 'm', m, 'n', n)
```

3 Description

The m by n matrix A is factorized as

$$A = \begin{pmatrix} R & 0 \end{pmatrix} P^{\mathrm{H}} \quad \text{when } m < n,$$

$$A = RP^{\mathrm{H}} \quad \text{when } m = n,$$

where P is an n by n unitary matrix and R is an m by m upper triangular matrix.

P is given as a sequence of Householder transformation matrices

$$P = P_m \cdots P_2 P_1,$$

the $(m - k + 1)$th transformation matrix, P_k, being used to introduce zeros into the kth row of A. P_k has the form

$$P_k = I - \gamma_k u_k u_k^H,$$

where

$$u_k = \begin{pmatrix} w_k \\ \zeta_k \\ 0 \\ z_k \end{pmatrix}.$$

γ_k is a scalar for which $\mathrm{Re}(\gamma_k) = 1.0$, ζ_k is a real scalar, w_k is a $(k - 1)$ element vector and z_k is an $(n - m)$ element vector. γ_k and u_k are chosen to annihilate the elements in the kth row of A.

The scalar γ_k and the vector u_k are returned in the kth element of **theta** and in the kth row of **a**, such that θ_k, given by

$$\theta_k = (\zeta_k, \mathrm{Im}(\gamma_k)).$$

is in **theta**(k), the elements of w_k are in $\mathbf{a}(k,1), \ldots, \mathbf{a}(k,k-1)$ and the elements of z_k are in $\mathbf{a}(k,m+1), \ldots, \mathbf{a}(k,n)$. The elements of R are returned in the upper triangular part of **a**.

4 References

Golub G H and Van Loan C F (1996) *Matrix Computations* (3rd Edition) Johns Hopkins University Press, Baltimore

Wilkinson J H (1965) *The Algebraic Eigenvalue Problem* Oxford University Press, Oxford

5 Parameters

5.1 Compulsory Input Parameters

1: **a**(lda, :) – COMPLEX (KIND=nag_wp) array

The first dimension of the array **a** must be at least max(1, **m**).

The second dimension of the array **a** must be at least max(1, **n**).

The leading m by n part of the array **a** must contain the matrix to be factorized.

5.2 Optional Input Parameters

1: **m** – INTEGER

Default: the first dimension of the array **a**.

m, the number of rows of the matrix A.

When **m** $= 0$ then an immediate return is effected.

Constraint: **m** ≥ 0.

2: **n** – INTEGER

Default: the second dimension of the array **a**.

n, the number of columns of the matrix A.

Constraint: **n** $\geq$ **m**.

5.3 Output Parameters

1: **a**(lda, :) – COMPLEX (KIND=nag_wp) array

The first dimension of the array **a** will be max(1, **m**).

The second dimension of the array **a** will be max(1, **n**).

The m by m upper triangular part of **a** will contain the upper triangular matrix R, and the m by m strictly lower triangular part of **a** and the m by $(n - m)$ rectangular part of **a** to the right of the upper triangular part will contain details of the factorization as described in Section 3.

2: **theta**(**m**) – COMPLEX (KIND=nag_wp) array

theta(k) contains the scalar θ_k for the $(m - k + 1)$th transformation. If $P_k = I$ then **theta**(k) $= 0.0$; if

$$T_k = \begin{pmatrix} I & 0 & 0 \\ 0 & \alpha & 0 \\ 0 & 0 & I \end{pmatrix}, \qquad \mathrm{Re}(\alpha) < 0.0$$

then **theta**(k) $= \alpha$, otherwise **theta**(k) contains θ_k as described in Section 3 and θ_k is always in the range $\left(1.0, \sqrt{2.0}\right)$.

3: **ifail** – INTEGER

ifail $= 0$ unless the function detects an error (see Section 5).

6 Error Indicators and Warnings

Errors or warnings detected by the function:

ifail $= -1$

On entry, **m** < 0,
or **n** $<$ **m**,
or $lda <$ **m**.

ifail $= -99$

An unexpected error has been triggered by this routine. Please contact NAG.

ifail $= -399$

Your licence key may have expired or may not have been installed correctly.

ifail $= -999$

Dynamic memory allocation failed.

7 Accuracy

The computed factors R and P satisfy the relation

$$(R0)P^{\mathrm{H}} = A + E,$$

where

$$\|E\| \leq c\epsilon\|A\|,$$

ϵ is the ***machine precision*** (see nag_machine_precision (x02aj)), c is a modest function of m and n, and $\|.\|$ denotes the spectral (two) norm.

8 Further Comments

The approximate number of floating-point operations is given by $8 \times m^2(3n - m)/3$.

The first k rows of the unitary matrix P^{H} can be obtained by calling nag_matop_complex_gen_rq_formq (f01rk), which overwrites the k rows of P^{H} on the first k rows of the array **a**. P^{H} is obtained by the call:

```
[a, ifail] = f01qk('Separate', m, k, a, theta);
```

9 Example

This example obtains the RQ factorization of the 3 by 5 matrix

$$A = \begin{pmatrix} -0.5i & 0.4 - 0.3i & 0.4 & 0.3 - 0.4i & 0.3i \\ -0.5 - 1.5i & 0.9 - 1.3i & -0.4 - 0.4i & 0.1 - 0.7i & 0.3 - 0.3i \\ -1.0 - 1.0i & 0.2 - 1.4i & 1.8 & 0.0 & -2.4i \end{pmatrix}.$$

9.1 Program Text

```
function f01rj_example

fprintf('f01rj example results\n\n');

a = [  0    - 0.5i,  0.4 - 0.3i,  0.4 + 0i,    0.3 + 0.4i,  0   + 0.3i;
      -0.5 - 1.5i,  0.9 - 1.3i, -0.4 - 0.4i,  0.1 - 0.7i,  0.3 - 0.3i;
      -1   - i,     0.2 - 1.4i,  1.8 + 0i,    0   + 0i,    0   - 2.4i];

[RQ, theta, ifail] = f01rj(a);
```

```
disp('RQ Factorization of A');
disp('Vector theta');
disp(theta');
disp('Matrix A after factorization (R in left-hand upper triangle');
disp(RQ);
```

9.2 Program Results

```
     f01rj example results

RQ Factorization of A
Vector theta
   1.0387 + 0.1006i   1.1810 - 0.3809i   1.2244 + 0.0000i

Matrix A after factorization (R in left-hand upper triangle
    0.7878 + 0.0000i  -0.2549 - 0.4006i  -0.2774 - 0.2774i  -0.2850 + 0.5586i
0.1154 + 0.7031i
    0.0396 + 0.5222i  -2.1122 + 0.0000i  -1.1094 - 0.5547i   0.1283 + 0.2317i
0.0790 - 0.0361i
   -0.2265 + 0.2265i   0.0453 + 0.3171i  -3.6056 + 0.0000i   0.0000 + 0.0000i
0.0000 + 0.5436i
```