NAG Toolbox

nag_det_real_band_sym (f03bh)

1 Purpose

nag_det_real_band_sym (f03bh) computes the determinant of a n by n symmetric positive definite banded matrix A that has been stored in band-symmetric storage. nag_lapack_dpbtrf (f07hd) must be called first to supply the Cholesky factorized form. The storage (upper or lower triangular) used by nag_lapack_dpbtrf (f07hd) is relevant as this determines which elements of the stored factorized form are referenced.

2 Syntax

```
[d, id, ifail] = nag_det_real_band_sym(uplo, kd, ab, 'n', n)
[d, id, ifail] = f03bh(uplo, kd, ab, 'n', n)
```

3 Description

The determinant of A is calculated using the Cholesky factorization $A = U^{\mathrm{T}}U$, where U is an upper triangular band matrix, or $A = LL^{\mathrm{T}}$, where L is a lower triangular band matrix. The determinant of A is the product of the squares of the diagonal elements of U or L.

4 References

Wilkinson J H and Reinsch C (1971) *Handbook for Automatic Computation II, Linear Algebra* Springer–Verlag

5 Parameters

5.1 Compulsory Input Parameters

1: **uplo** – CHARACTER(1)

Indicates whether the upper or lower triangular part of A was stored and how it was factorized. This should not be altered following a call to nag_lapack_dpbtrf (f07hd).

uplo = 'U'
The upper triangular part of A was originally stored and A was factorized as $U^{\mathrm{T}}U$ where U is upper triangular.

uplo = 'L'
The lower triangular part of A was originally stored and A was factorized as LL^{T} where L is lower triangular.

Constraint: **uplo** = 'U' or 'L'.

2: **kd** – INTEGER

k_d, the number of superdiagonals or subdiagonals of the matrix A.

Constraint: **kd** ≥ 0.

3: **ab**$(ldab, :)$ – REAL (KIND=nag_wp) array

The first dimension of the array **ab** must be at least **kd** $+ 1$.

The second dimension of the array **ab** must be at least $\max(1, \mathbf{n})$.

The Cholesky factor of A, as returned by nag_lapack_dpbtrf (f07hd).

5.2 Optional Input Parameters

1: **n** – INTEGER

Default: the second dimension of the array **ab**.

n, the order of the matrix A.

Constraint: $\mathbf{n} > 0$.

5.3 Output Parameters

1: **d** – REAL (KIND=nag_wp)

2: **id** – INTEGER

The determinant of A is given by $\mathbf{d} \times 2.0^{\mathbf{id}}$. It is given in this form to avoid overflow or underflow.

3: **ifail** – INTEGER

$\mathbf{ifail} = 0$ unless the function detects an error (see Section 5).

6 Error Indicators and Warnings

Errors or warnings detected by the function:

$\mathbf{ifail} = 1$

Constraint: $\mathbf{uplo} =$ 'L' or 'U'.

$\mathbf{ifail} = 2$

Constraint: $\mathbf{n} > 0$.

$\mathbf{ifail} = 3$

Constraint: $\mathbf{kd} \geq 0$.

$\mathbf{ifail} = 5$

Constraint: $ldab \geq \mathbf{kd} + 1$.

$\mathbf{ifail} = 6$

The matrix A is not positive definite.

$\mathbf{ifail} = -99$

An unexpected error has been triggered by this routine. Please contact NAG.

$\mathbf{ifail} = -399$

Your licence key may have expired or may not have been installed correctly.

$\mathbf{ifail} = -999$

Dynamic memory allocation failed.

7 Accuracy

The accuracy of the determinant depends on the conditioning of the original matrix. For a detailed error analysis see page 54 of Wilkinson and Reinsch (1971).

8 Further Comments

The time taken by nag_det_real_band_sym (f03bh) is approximately proportional to n.

This function should only be used when $m \ll n$ since as m approaches n, it becomes less efficient to take advantage of the band form.

9 Example

This example calculates the determinant of the real symmetric positive definite band matrix

$$\begin{pmatrix} 5 & -4 & 1 & & & & \\ -4 & 6 & -4 & 1 & & & \\ 1 & -4 & 6 & -4 & 1 & & \\ & 1 & -4 & 6 & -4 & 1 & \\ & & 1 & -4 & 6 & -4 & 1 \\ & & & 1 & -4 & 6 & -4 \\ & & & & 1 & -4 & 5 \end{pmatrix}.$$

9.1 Program Text

```
     function f03bh_example

fprintf('f03bh example results\n\n');

uplo = 'l';
kd   = nag_int(2);
n    = nag_int(7);
ab = [ 5,  6,  6,  6,  6,  6,  5;
      -4, -4, -4, -4, -4, -4,  0;
       1,  1,  1,  1,  1,  0,  0];
% Factorize a
[ab, info] = f07hd(uplo, kd, ab);

if info == 0
  fprintf('\n');
  [ifail] = x04ce(n, n, kd, nag_int(0), ab, 'Array ab after factorization');

  [d, id, ifail] = f03bh(uplo, kd, ab);

  fprintf('d = %13.5f id = %d\n', d, id);
  fprintf('Value of determinant = %13.5e\n', d*2^id);
else
  fprintf('\n** Factorization routine returned error flag info = %d\n', info);
end
```

9.2 Program Results

```
     f03bh example results

 Array ab after factorization
            1          2          3          4          5          6          7
 1      2.2361
 2     -1.7889     1.6733
 3      0.4472    -1.9124     1.4639
 4                 0.5976    -1.9518     1.3540
 5                           0.6831    -1.9695     1.2863
 6                                      0.7385    -1.9789     1.2403
 7                                                0.7774    -1.9846     0.6761
d =       0.25000 id = 8
Value of determinant =   6.40000e+01
```