NAG Toolbox

nag_lapack_ztrcon (f07tu)

1 Purpose

nag_lapack_ztrcon (f07tu) estimates the condition number of a complex triangular matrix.

2 Syntax

```
[rcond, info] = nag_lapack_ztrcon(norm_p, uplo, diag, a, 'n', n)
[rcond, info] = f07tu(norm_p, uplo, diag, a, 'n', n)
```

3 Description

nag_lapack_ztrcon (f07tu) estimates the condition number of a complex triangular matrix A, in either the 1-norm or the ∞-norm:

$$\kappa_1(A) = \|A\|_1 \left\|A^{-1}\right\|_1 \quad \text{or} \quad \kappa_\infty(A) = \|A\|_\infty \left\|A^{-1}\right\|_\infty.$$

Note that $\kappa_\infty(A) = \kappa_1(A^{\mathrm{T}})$.

Because the condition number is infinite if A is singular, the function actually returns an estimate of the **reciprocal** of the condition number.

The function computes $\|A\|_1$ or $\|A\|_\infty$ exactly, and uses Higham's implementation of Hager's method (see Higham (1988)) to estimate $\left\|A^{-1}\right\|_1$ or $\left\|A^{-1}\right\|_\infty$.

4 References

Higham N J (1988) FORTRAN codes for estimating the one-norm of a real or complex matrix, with applications to condition estimation *ACM Trans. Math. Software* **14** 381–396

5 Parameters

5.1 Compulsory Input Parameters

1: **norm_p** – CHARACTER(1)

Indicates whether $\kappa_1(A)$ or $\kappa_\infty(A)$ is estimated.

norm_p = '1' or 'O'
$\kappa_1(A)$ is estimated.

norm_p = 'I'
$\kappa_\infty(A)$ is estimated.

Constraint: **norm_p** = '1', 'O' or 'I'.

2: **uplo** – CHARACTER(1)

Specifies whether A is upper or lower triangular.

uplo = 'U'
A is upper triangular.

uplo = 'L'
A is lower triangular.

Constraint: **uplo** = 'U' or 'L'.

3: **diag** – CHARACTER(1)

Indicates whether A is a nonunit or unit triangular matrix.

diag = 'N'
A is a nonunit triangular matrix.

diag = 'U'
A is a unit triangular matrix; the diagonal elements are not referenced and are assumed to be 1.

Constraint: **diag** = 'N' or 'U'.

4: **a**$(lda,:)$ – COMPLEX (KIND=nag_wp) array

The first dimension of the array **a** must be at least $\max(1,\mathbf{n})$.

The second dimension of the array **a** must be at least $\max(1,\mathbf{n})$.

The n by n triangular matrix A.

If **uplo** = 'U', a is upper triangular and the elements of the array below the diagonal are not referenced.

If **uplo** = 'L', a is lower triangular and the elements of the array above the diagonal are not referenced.

If **diag** = 'U', the diagonal elements of a are assumed to be 1, and are not referenced.

5.2 Optional Input Parameters

1: **n** – INTEGER

Default: the first dimension of the array **a** and the second dimension of the array **a**.

n, the order of the matrix A.

Constraint: $\mathbf{n} \geq 0$.

5.3 Output Parameters

1: **rcond** – REAL (KIND=nag_wp)

An estimate of the reciprocal of the condition number of A. **rcond** is set to zero if exact singularity is detected or the estimate underflows. If **rcond** is less than ***machine precision***, A is singular to working precision.

2: **info** – INTEGER

info = 0 unless the function detects an error (see Section 6).

6 Error Indicators and Warnings

info < 0

If **info** $= -i$, argument i had an illegal value. An explanatory message is output, and execution of the program is terminated.

7 Accuracy

The computed estimate **rcond** is never less than the true value ρ, and in practice is nearly always less than 10ρ, although examples can be constructed where **rcond** is much larger.

8 Further Comments

A call to nag_lapack_ztrcon (f07tu) involves solving a number of systems of linear equations of the form $Ax = b$ or $A^{\mathrm{H}}x = b$; the number is usually 5 and never more than 11. Each solution involves approximately $4n^2$ real floating-point operations but takes considerably longer than a call to nag_lapack_ztrtrs (f07ts) with one right-hand side, because extra care is taken to avoid overflow when A is approximately singular.

The real analogue of this function is nag_lapack_dtrcon (f07tg).

9 Example

This example estimates the condition number in the 1-norm of the matrix A, where

$$A = \begin{pmatrix} 4.78 + 4.56i & 0.00 + 0.00i & 0.00 + 0.00i & 0.00 + 0.00i \\ 2.00 - 0.30i & -4.11 + 1.25i & 0.00 + 0.00i & 0.00 + 0.00i \\ 2.89 - 1.34i & 2.36 - 4.25i & 4.15 + 0.80i & 0.00 + 0.00i \\ -1.89 + 1.15i & 0.04 - 3.69i & -0.02 + 0.46i & 0.33 - 0.26i \end{pmatrix}.$$

The true condition number in the 1-norm is 70.27.

9.1 Program Text

```
     function f07tu_example

fprintf('f07tu example results\n\n');

% Estimate condition number of A, where A is Lower triangular
a = [  4.78 +  4.56i,   0    +  0i,     0    + 0i,    0    + 0i;
       2.00 -  0.30i,  -4.11 +  1.25i,  0    + 0i,    0    + 0i;
       2.89 -  1.34i,   2.36 -  4.25i,  4.15 + 0.8i,  0    + 0i;
      -1.89 +  1.15i,   0.04 -  3.69i, -0.02 + 0.46i, 0.33 - 0.26i];

% Get reciprocal condition number
norm_p = '1';
uplo = 'L';
diag = 'N';
[rcond, info] = f07tu( ...
                       norm_p, uplo, diag, a);

fprintf('Estimate of condition number = %9.2e\n', 1/rcond);
```

9.2 Program Results

```
     f07tu example results

Estimate of condition number =  3.74e+01
```