

NAG Toolbox

nag_lapack_dorglq (f08aj)

1 Purpose

nag_lapack_dorglq (f08aj) generates all or part of the real orthogonal matrix Q from an LQ factorization computed by nag_lapack_dgelqf (f08ah).

2 Syntax

```
[a, info] = nag_lapack_dorglq(a, tau, 'm', m, 'n', n, 'k', k)
[a, info] = f08aj(a, tau, 'm', m, 'n', n, 'k', k)
```

3 Description

nag_lapack_dorglq (f08aj) is intended to be used after a call to nag_lapack_dgelqf (f08ah), which performs an LQ factorization of a real matrix A . The orthogonal matrix Q is represented as a product of elementary reflectors.

This function may be used to generate Q explicitly as a square matrix, or to form only its leading rows. Usually Q is determined from the LQ factorization of a p by n matrix A with $p \leq n$. The whole of Q may be computed by:

```
[a, info] = f08aj(a, tau);
```

(note that the array **a** must have at least n rows) or its leading p rows by:

```
[a, info] = f08aj(a(1:p,:), tau);
```

The rows of Q returned by the last call form an orthonormal basis for the space spanned by the rows of A ; thus nag_lapack_dgelqf (f08ah) followed by nag_lapack_dorglq (f08aj) can be used to orthogonalize the rows of A .

The information returned by the LQ factorization functions also yields the LQ factorization of the leading k rows of A , where $k < p$. The orthogonal matrix arising from this factorization can be computed by:

```
[a, info] = f08aj(a, tau);
```

or its leading k rows by:

```
[a, info] = f08aj(a(1:k,:), tau);
```

4 References

Golub G H and Van Loan C F (1996) *Matrix Computations* (3rd Edition) Johns Hopkins University Press, Baltimore

5 Parameters

5.1 Compulsory Input Parameters

1: **a**(*lda*,:) – REAL (KIND=nag_wp) array

The first dimension of the array **a** must be at least max(1, **m**).

The second dimension of the array **a** must be at least max(1, **n**).

Details of the vectors which define the elementary reflectors, as returned by nag_lapack_dgelqf (f08ah).

2: **tau**(:) – REAL (KIND=nag_wp) array

The dimension of the array **tau** must be at least $\max(1, \mathbf{k})$

Further details of the elementary reflectors, as returned by nag_lapack_dgels (f08ah).

5.2 Optional Input Parameters

1: **m** – INTEGER

Default: the first dimension of the array **a**.

m , the number of rows of the matrix Q .

Constraint: $\mathbf{m} \geq 0$.

2: **n** – INTEGER

Default: the second dimension of the array **a**.

n , the number of columns of the matrix Q .

Constraint: $\mathbf{n} \geq \mathbf{m}$.

3: **k** – INTEGER

Default: the dimension of the array **tau**.

k , the number of elementary reflectors whose product defines the matrix Q .

Constraint: $\mathbf{m} \geq \mathbf{k} \geq 0$.

5.3 Output Parameters

1: **a**(*lda*, :) – REAL (KIND=nag_wp) array

The first dimension of the array **a** will be $\max(1, \mathbf{m})$.

The second dimension of the array **a** will be $\max(1, \mathbf{n})$.

The m by n matrix Q .

2: **info** – INTEGER

info = 0 unless the function detects an error (see Section 6).

6 Error Indicators and Warnings

info = $-i$

If **info** = $-i$, parameter i had an illegal value on entry. The parameters are numbered as follows:

1: **m**, 2: **n**, 3: **k**, 4: **a**, 5: **lda**, 6: **tau**, 7: **work**, 8: **lwork**, 9: **info**.

It is possible that **info** refers to a parameter that is omitted from the MATLAB interface. This usually indicates that an error in one of the other input parameters has caused an incorrect value to be inferred.

7 Accuracy

The computed matrix Q differs from an exactly orthogonal matrix by a matrix E such that

$$\|E\|_2 = O(\epsilon),$$

where ϵ is the *machine precision*.

8 Further Comments

The total number of floating-point operations is approximately $4mnk - 2(m + n)k^2 + \frac{4}{3}k^3$; when $m = k$, the number is approximately $\frac{2}{3}m^2(3n - m)$.

The complex analogue of this function is nag_lapack_zunglq (f08aw).

9 Example

This example forms the leading 4 rows of the orthogonal matrix Q from the LQ factorization of the matrix A , where

$$A = \begin{pmatrix} -5.42 & 3.28 & -3.68 & 0.27 & 2.06 & 0.46 \\ -1.65 & -3.40 & -3.20 & -1.03 & -4.06 & -0.01 \\ -0.37 & 2.35 & 1.90 & 4.31 & -1.76 & 1.13 \\ -3.15 & -0.11 & 1.99 & -2.70 & 0.26 & 4.50 \end{pmatrix}.$$

The rows of Q form an orthonormal basis for the space spanned by the rows of A .

9.1 Program Text

```
function f08aj_example

fprintf('f08aj example results\n\n');

a = [ -5.42   3.28  -3.68   0.27   2.06   0.46;
      -1.65  -3.40  -3.20  -1.03  -4.06  -0.01;
       -0.37   2.35   1.90   4.31  -1.76   1.13;
      -3.15  -0.11   1.99  -2.70   0.26   4.50];

% Compute the LQ Factorisation of A
[lq, tau, info] = f08ah(a);

% Generate Q
[q, info] = f08aj(lq, tau);

disp('Orthogonal factor Q');
disp(q);
```

9.2 Program Results

```
f08aj example results

Orthogonal factor Q
-0.7104    0.4299   -0.4824    0.0354    0.2700    0.0603
-0.2412   -0.5323   -0.4845   -0.1595   -0.6311   -0.0027
 0.1287   -0.2619   -0.2108   -0.7447    0.5227   -0.2063
-0.3403   -0.0921    0.4546   -0.3869   -0.0465    0.7191
```
