NAG Toolbox

nag_lapack_dorgrq (f08cj)

1 Purpose

nag_lapack_dorgrq (f08cj) generates all or part of the real n by n orthogonal matrix Q from an RQ factorization computed by nag_lapack_dgerqf (f08ch).

2 Syntax

```
[a, info] = nag_lapack_dorgrq(a, tau, 'm', m, 'n', n, 'k', k)
[a, info] = f08cj(a, tau, 'm', m, 'n', n, 'k', k)
```

3 Description

nag_lapack_dorgrq (f08cj) is intended to be used following a call to nag_lapack_dgerqf (f08ch), which performs an RQ factorization of a real matrix A and represents the orthogonal matrix Q as a product of k elementary reflectors of order n.

This function may be used to generate Q explicitly as a square matrix, or to form only its trailing rows.

Usually Q is determined from the RQ factorization of a p by n matrix A with $p \leq n$. The whole of Q may be computed by:

```
[a, info] = f08cj(a, tau);
```

(note that the matrix A must have at least n rows), or its trailing p rows as:

```
[a, info] = f08cj(a(1:p,:), tau, 'k', p);
```

The rows of Q returned by the last call form an orthonormal basis for the space spanned by the rows of A; thus nag_lapack_dgerqf (f08ch) followed by nag_lapack_dorgrq (f08cj) can be used to orthogonalize the rows of A.

The information returned by nag_lapack_dgerqf (f08ch) also yields the RQ factorization of the trailing k rows of A, where $k < p$. The orthogonal matrix arising from this factorization can be computed by:

```
[a, info] = f08cj(a, tau, 'k', k);
```

or its leading k columns by:

```
[a, info] = f08cj(a(1:k,:), tau, 'k', k);
```

4 References

Anderson E, Bai Z, Bischof C, Blackford S, Demmel J, Dongarra J J, Du Croz J J, Greenbaum A, Hammarling S, McKenney A and Sorensen D (1999) *LAPACK Users' Guide* (3rd Edition) SIAM, Philadelphia http://www.netlib.org/lapack/lug

Golub G H and Van Loan C F (1996) *Matrix Computations* (3rd Edition) Johns Hopkins University Press, Baltimore

5 Parameters

5.1 Compulsory Input Parameters

1: **a**(lda,:) – REAL (KIND=nag_wp) array

The first dimension of the array **a** must be at least $\max(1, \mathbf{m})$.

The second dimension of the array **a** must be at least $\max(1, \mathbf{n})$.

Details of the vectors which define the elementary reflectors, as returned by nag_lapack_dgerqf (f08ch).

2: **tau**(:) – REAL (KIND=nag_wp) array

The dimension of the array **tau** must be at least $\max(1, \mathbf{k})$

tau(i) must contain the scalar factor of the elementary reflector H_i, as returned by nag_lapack_dgerqf (f08ch).

5.2 Optional Input Parameters

1: **m** – INTEGER

Default: the first dimension of the array **a**.

m, the number of rows of the matrix Q.

Constraint: $\mathbf{m} \geq 0$.

2: **n** – INTEGER

Default: the second dimension of the array **a**.

n, the number of columns of the matrix Q.

Constraint: $\mathbf{n} \geq \mathbf{m}$.

3: **k** – INTEGER

Default: the dimension of the array **tau**.

k, the number of elementary reflectors whose product defines the matrix Q.

Constraint: $\mathbf{m} \geq \mathbf{k} \geq 0$.

5.3 Output Parameters

1: **a**$(lda,:)$ – REAL (KIND=nag_wp) array

The first dimension of the array **a** will be $\max(1, \mathbf{m})$.

The second dimension of the array **a** will be $\max(1, \mathbf{n})$.

The m by n matrix Q.

2: **info** – INTEGER

info $= 0$ unless the function detects an error (see Section 6).

6 Error Indicators and Warnings

info $= -i$

If **info** $= -i$, parameter i had an illegal value on entry. The parameters are numbered as follows:

1: **m**, 2: **n**, 3: **k**, 4: **a**, 5: **lda**, 6: **tau**, 7: **work**, 8: **lwork**, 9: **info**.

It is possible that **info** refers to a parameter that is omitted from the MATLAB interface. This usually indicates that an error in one of the other input parameters has caused an incorrect value to be inferred.

7 Accuracy

The computed matrix Q differs from an exactly orthogonal matrix by a matrix E such that

$$\|E\|_2 = O\epsilon$$

and ϵ is the ***machine precision***.

8 Further Comments

The total number of floating-point operations is approximately $4mnk - 2(m+n)k^2 + \frac{4}{3}k^3$; when $m = k$ this becomes $\frac{2}{3}m^2(3n - m)$.

The complex analogue of this function is nag_lapack_zungrq (f08cw).

9 Example

This example generates the first four rows of the matrix Q of the RQ factorization of A as returned by nag_lapack_dgerqf (f08ch), where

$$A = \begin{pmatrix} -0.57 & -1.93 & 2.30 & -1.93 & 0.15 & -0.02 \\ -1.28 & 1.08 & 0.24 & 0.64 & 0.30 & 1.03 \\ -0.39 & -0.31 & 0.40 & -0.66 & 0.15 & -1.43 \\ 0.25 & -2.14 & -0.35 & 0.08 & -2.13 & 0.50 \end{pmatrix}.$$

Note that the block size (NB) of 64 assumed in this example is not realistic for such a small problem, but should be suitable for large problems.

9.1 Program Text

```
     function f08cj_example

fprintf('f08cj example results\n\n');

a = [-0.57  -1.93   2.30  -1.93   0.15  -0.02;
     -1.28   1.08   0.24   0.64   0.30   1.03;
     -0.39  -0.31   0.40  -0.66   0.15  -1.43;
      0.25  -2.14  -0.35   0.08  -2.13   0.50];

% Compute the RQ Factorisation of A
[rq, tau, info] = f08ch(a);

% Form Q
[Q, info] = f08cj(rq, tau);

disp('Orthogonal factor Q');
disp(Q);
```

9.2 Program Results

```
     f08cj example results

Orthogonal factor Q
   -0.0833    0.2972   -0.6404    0.4461   -0.2938   -0.4575
    0.9100   -0.1080   -0.2351   -0.1620    0.2022   -0.1946
   -0.2202   -0.2706    0.2220   -0.3866    0.0015   -0.8243
   -0.0809    0.6922    0.1132   -0.0259    0.6890   -0.1617
```