NAG Toolbox

nag_lapack_zgerqf (f08cv)

1 Purpose

nag_lapack_zgerqf (f08cv) computes an RQ factorization of a complex m by n matrix A.

2 Syntax

```
[a, tau, info] = nag_lapack_zgerqf(a, 'm', m, 'n', n)
[a, tau, info] = f08cv(a, 'm', m, 'n', n)
```

3 Description

nag_lapack_zgerqf (f08cv) forms the RQ factorization of an arbitrary rectangular real m by n matrix. If $m \leq n$, the factorization is given by

$$A = \begin{pmatrix} 0 & R \end{pmatrix} Q,$$

where R is an m by m lower triangular matrix and Q is an n by n unitary matrix. If $m > n$ the factorization is given by

$$A = RQ,$$

where R is an m by n upper trapezoidal matrix and Q is again an n by n unitary matrix. In the case where $m < n$ the factorization can be expressed as

$$A = \begin{pmatrix} 0 & R \end{pmatrix} \begin{pmatrix} Q_1 \\ Q_2 \end{pmatrix} = RQ_2,$$

where Q_1 consists of the first $(n - m)$ rows of Q and Q_2 the remaining m rows.

The matrix Q is not formed explicitly, but is represented as a product of $\min(m, n)$ elementary reflectors (see the F08 Chapter Introduction for details). Functions are provided to work with Q in this representation (see Section 9).

4 References

Anderson E, Bai Z, Bischof C, Blackford S, Demmel J, Dongarra J J, Du Croz J J, Greenbaum A, Hammarling S, McKenney A and Sorensen D (1999) *LAPACK Users' Guide* (3rd Edition) SIAM, Philadelphia http://www.netlib.org/lapack/lug

Golub G H and Van Loan C F (1996) *Matrix Computations* (3rd Edition) Johns Hopkins University Press, Baltimore

5 Parameters

5.1 Compulsory Input Parameters

1: **a**(lda, :) – COMPLEX (KIND=nag_wp) array

The first dimension of the array **a** must be at least $\max(1, \mathbf{m})$.

The second dimension of the array **a** must be at least $\max(1, \mathbf{n})$.

The m by n matrix A.

5.2 Optional Input Parameters

1: **m** – INTEGER

Default: the first dimension of the array **a**.

m, the number of rows of the matrix A.

Constraint: $\mathbf{m} \geq 0$.

2: **n** – INTEGER

Default: the second dimension of the array **a**.

n, the number of columns of the matrix A.

Constraint: $\mathbf{n} \geq 0$.

5.3 Output Parameters

1: $\mathbf{a}(lda,:)$ – COMPLEX (KIND=nag_wp) array

The first dimension of the array **a** will be $\max(1,\mathbf{m})$.

The second dimension of the array **a** will be $\max(1,\mathbf{n})$.

If $m \leq n$, the upper triangle of the subarray $\mathbf{a}(1:m, n-m+1:n)$ contains the m by m upper triangular matrix R.

If $m \geq n$, the elements on and above the $(m-n)$th subdiagonal contain the m by n upper trapezoidal matrix R; the remaining elements, with the array **tau**, represent the unitary matrix Q as a product of $\min(m,n)$ elementary reflectors (see Section 3.2.6 in the F08 Chapter Introduction).

2: $\mathbf{tau}(:)$ – COMPLEX (KIND=nag_wp) array

The dimension of the array **tau** will be $\max(1,\min(\mathbf{m},\mathbf{n}))$

The scalar factors of the elementary reflectors.

3: **info** – INTEGER

$\mathbf{info} = 0$ unless the function detects an error (see Section 6).

6 Error Indicators and Warnings

$\mathbf{info} = -i$

If $\mathbf{info} = -i$, parameter i had an illegal value on entry. The parameters are numbered as follows:

1: **m**, 2: **n**, 3: **a**, 4: **lda**, 5: **tau**, 6: **work**, 7: **lwork**, 8: **info**.

It is possible that **info** refers to a parameter that is omitted from the MATLAB interface. This usually indicates that an error in one of the other input parameters has caused an incorrect value to be inferred.

7 Accuracy

The computed factorization is the exact factorization of a nearby matrix $A+E$, where

$$\|E\|_2 = O\epsilon\|A\|_2$$

and ϵ is the ***machine precision***.

8 Further Comments

The total number of floating-point operations is approximately $\frac{2}{3}m^2(3n - m)$ if $m \leq n$, or $\frac{2}{3}n^2(3m - n)$ if $m > n$.

To form the unitary matrix Q nag_lapack_zgerqf (f08cv) may be followed by a call to nag_lapack_zungrq (f08cw):

```
[a, info] = f08cw(a, tau, 'k', min(m,n));
```

but note that the first dimension of the array **a** must be at least **n**, which may be larger than was required by nag_lapack_zgerqf (f08cv). When $m \leq n$, it is often only the first m rows of Q that are required and they may be formed by the call:

```
[a, info] = f08cw(a, tau);
```

To apply Q to an arbitrary real rectangular matrix C, nag_lapack_zgerqf (f08cv) may be followed by a call to nag_lapack_zunmrq (f08cx). For example:

```
[a, c, info] = f08cx('Left','C', a, tau, c);
```

forms $C = Q^{\mathrm{H}}C$, where C is n by p.

The real analogue of this function is nag_lapack_dgerqf (f08ch).

9 Example

This example finds the minimum norm solution to the underdetermined equations

$$Ax = b$$

where

$$A = \begin{pmatrix} 0.28 - 0.36i & 0.50 - 0.86i & -0.77 - 0.48i & 1.58 + 0.66i \\ -0.50 - 1.10i & -1.21 + 0.76i & -0.32 - 0.24i & -0.27 - 1.15i \\ 0.36 - 0.51i & -0.07 + 1.33i & -0.75 + 0.47i & -0.08 + 1.01i \end{pmatrix}$$

and

$$b = \begin{pmatrix} -1.35 + 0.19i \\ 9.41 - 3.56i \\ -7.57 + 6.93i \end{pmatrix}.$$

The solution is obtained by first obtaining an RQ factorization of the matrix A.

Note that the block size (NB) of 64 assumed in this example is not realistic for such a small problem, but should be suitable for large problems.

9.1 Program Text

```
     function f08cv_example

fprintf('f08cv example results\n\n');

% Minimum norm solution of AX = B, m<n
m = 3;
n = 4;
a = [ 0.28 - 0.36i,  0.50 - 0.86i, -0.77 - 0.48i,  1.58 + 0.66i;
     -0.50 - 1.10i, -1.21 + 0.76i, -0.32 - 0.24i, -0.27 - 1.15i;
      0.36 - 0.51i, -0.07 + 1.33i, -0.75 + 0.47i, -0.08 + 1.01i];
b = [ -1.35 + 0.19i;
       9.41 - 3.56i;
      -7.57 + 6.93i];

% Compute the RQ factorization of A
[rq, tau, info] = f08cv(a);

% RQX = B ==> C = QX = R^-1 B
c = zeros(n, 1);
```

```
il = n - m + 1;
[c(il:n,:), info] = f07ts( ...
    'Upper', 'No transpose','Non-Unit', rq(:,il:n), b);

% QX = C ==> X = Q^H C
[rq, x, info] = f08cx( ...
        'Left', 'Conjugate Transpose', rq, tau, c);

fprintf('Minimum-norm solution\n');
disp(x);
```

9.2 Program Results

```
     f08cv example results

Minimum-norm solution
  -2.8501 + 6.4683i
   1.6264 - 0.7799i
   6.9290 + 4.6481i
   1.4048 + 3.2400i
```