

NAG Toolbox

nag_lapack_dsyevr (f08fd)

1 Purpose

nag_lapack_dsyevr (f08fd) computes selected eigenvalues and, optionally, eigenvectors of a real n by n symmetric matrix A . Eigenvalues and eigenvectors can be selected by specifying either a range of values or a range of indices for the desired eigenvalues.

2 Syntax

```
[a, m, w, z, isuppz, info] = nag_lapack_dsyevr(jobz, range, uplo, a, vl, vu, il,
iu, abstol, 'n', n)
```

```
[a, m, w, z, isuppz, info] = f08fd(jobz, range, uplo, a, vl, vu, il, iu, abstol,
'n', n)
```

3 Description

The symmetric matrix is first reduced to a tridiagonal matrix T , using orthogonal similarity transformations. Then whenever possible, nag_lapack_dsyevr (f08fd) computes the eigenspectrum using Relatively Robust Representations. nag_lapack_dsyevr (f08fd) computes eigenvalues by the dqds algorithm, while orthogonal eigenvectors are computed from various ‘good’ LDL^T representations (also known as Relatively Robust Representations). Gram–Schmidt orthogonalization is avoided as far as possible. More specifically, the various steps of the algorithm are as follows. For the i th unreduced block of T :

- compute $T - \sigma_i I = L_i D_i L_i^T$, such that $L_i D_i L_i^T$ is a relatively robust representation,
- compute the eigenvalues, λ_j , of $L_i D_i L_i^T$ to high relative accuracy by the dqds algorithm,
- if there is a cluster of close eigenvalues, ‘choose’ σ_i close to the cluster, and go to (a),
- given the approximate eigenvalue λ_j of $L_i D_i L_i^T$, compute the corresponding eigenvector by forming a rank-revealing twisted factorization.

The desired accuracy of the output can be specified by the argument **abstol**. For more details, see Dhillon (1997) and Parlett and Dhillon (2000).

4 References

Anderson E, Bai Z, Bischof C, Blackford S, Demmel J, Dongarra J J, Du Croz J J, Greenbaum A, Hammarling S, McKenney A and Sorensen D (1999) *LAPACK Users' Guide* (3rd Edition) SIAM, Philadelphia <http://www.netlib.org/lapack/lug>

Barlow J and Demmel J W (1990) Computing accurate eigensystems of scaled diagonally dominant matrices *SIAM J. Numer. Anal.* **27** 762–791

Demmel J W and Kahan W (1990) Accurate singular values of bidiagonal matrices *SIAM J. Sci. Statist. Comput.* **11** 873–912

Dhillon I (1997) A new $O(n^2)$ algorithm for the symmetric tridiagonal eigenvalue/eigenvector problem *Computer Science Division Technical Report No. UCB//CSD-97-971* UC Berkeley

Golub G H and Van Loan C F (1996) *Matrix Computations* (3rd Edition) Johns Hopkins University Press, Baltimore

Parlett B N and Dhillon I S (2000) Relatively robust representations of symmetric tridiagonals *Linear Algebra Appl.* **309** 121–151

5 Parameters

5.1 Compulsory Input Parameters

1: **jobz** – CHARACTER(1)

Indicates whether eigenvectors are computed.

jobz = 'N'

Only eigenvalues are computed.

jobz = 'V'

Eigenvalues and eigenvectors are computed.

Constraint: **jobz** = 'N' or 'V'.

2: **range** – CHARACTER(1)

If **range** = 'A', all eigenvalues will be found.

If **range** = 'V', all eigenvalues in the half-open interval (**vl**, **vu**] will be found.

If **range** = 'I', the **il**th to **iu**th eigenvalues will be found.

For **range** = 'V' or 'I' and **iu** – **il** < **n** – 1, nag_lapack_dstebz (f08jj) and nag_lapack_dstein (f08jk) are called.

Constraint: **range** = 'A', 'V' or 'I'.

3: **uplo** – CHARACTER(1)

If **uplo** = 'U', the upper triangular part of *A* is stored.

If **uplo** = 'L', the lower triangular part of *A* is stored.

Constraint: **uplo** = 'U' or 'L'.

4: **a**(*lda*,:) – REAL (KIND=nag_wp) array

The first dimension of the array **a** must be at least max(1, **n**).

The second dimension of the array **a** must be at least max(1, **n**).

The *n* by *n* symmetric matrix *A*.

If **uplo** = 'U', the upper triangular part of *a* must be stored and the elements of the array below the diagonal are not referenced.

If **uplo** = 'L', the lower triangular part of *a* must be stored and the elements of the array above the diagonal are not referenced.

5: **vl** – REAL (KIND=nag_wp)

6: **vu** – REAL (KIND=nag_wp)

If **range** = 'V', the lower and upper bounds of the interval to be searched for eigenvalues.

If **range** = 'A' or 'I', **vl** and **vu** are not referenced.

Constraint: if **range** = 'V', **vl** < **vu**.

7: **il** – INTEGER

8: **iu** – INTEGER

If **range** = 'I', the indices (in ascending order) of the smallest and largest eigenvalues to be returned.

If **range** = 'A' or 'V', **il** and **iu** are not referenced.

Constraints:

if **range** = 'I' and **n** = 0, **il** = 1 and **iu** = 0;
 if **range** = 'I' and **n** > 0, $1 \leq \mathbf{il} \leq \mathbf{iu} \leq \mathbf{n}$.

9: **abstol** – REAL (KIND=nag_wp)

The absolute error tolerance for the eigenvalues. An approximate eigenvalue is accepted as converged when it is determined to lie in an interval $[a, b]$ of width less than or equal to

$$\mathbf{abstol} + \epsilon \max(|a|, |b|),$$

where ϵ is the *machine precision*. If **abstol** is less than or equal to zero, then $\epsilon \|T\|_1$ will be used in its place, where T is the tridiagonal matrix obtained by reducing A to tridiagonal form. See Demmel and Kahan (1990).

If high relative accuracy is important, set **abstol** to `x02am()`, although doing so does not currently guarantee that eigenvalues are computed to high relative accuracy. See Barlow and Demmel (1990) for a discussion of which matrices can define their eigenvalues to high relative accuracy.

5.2 Optional Input Parameters

1: **n** – INTEGER

Default: the first dimension of the array **a** and the second dimension of the array **a**. (An error is raised if these dimensions are not equal.)

n , the order of the matrix A .

Constraint: $\mathbf{n} \geq 0$.

5.3 Output Parameters

1: **a(lda,:)** – REAL (KIND=nag_wp) array

The first dimension of the array **a** will be $\max(1, \mathbf{n})$.

The second dimension of the array **a** will be $\max(1, \mathbf{n})$.

The lower triangle (if **uplo** = 'L') or the upper triangle (if **uplo** = 'U') of **a**, including the diagonal, is overwritten.

2: **m** – INTEGER

The total number of eigenvalues found. $0 \leq \mathbf{m} \leq \mathbf{n}$.

If **range** = 'A', $\mathbf{m} = \mathbf{n}$.

If **range** = 'I', $\mathbf{m} = \mathbf{iu} - \mathbf{il} + 1$.

3: **w(:)** – REAL (KIND=nag_wp) array

The dimension of the array **w** will be $\max(1, \mathbf{n})$

The first \mathbf{m} elements contain the selected eigenvalues in ascending order.

4: **z(ldz,:)** – REAL (KIND=nag_wp) array

The first dimension, ldz , of the array **z** will be

if **jobz** = 'V', $ldz = \max(1, \mathbf{n})$;
 otherwise $ldz = 1$.

The second dimension of the array **z** will be $\max(1, \mathbf{m})$ if **jobz** = 'V' and 1 otherwise.

If **jobz** = 'V', the first **m** columns of Z contain the orthonormal eigenvectors of the matrix A corresponding to the selected eigenvalues, with the i th column of Z holding the eigenvector associated with $\mathbf{w}(i)$.

If **jobz** = 'N', \mathbf{z} is not referenced.

5: **isuppz**(:) – INTEGER array

The dimension of the array **isuppz** will be $\max(1, 2 \times \mathbf{m})$

The support of the eigenvectors in \mathbf{z} , i.e., the indices indicating the nonzero elements in \mathbf{z} . The i th eigenvector is nonzero only in elements **isuppz**($2 \times i - 1$) through **isuppz**($2 \times i$). Implemented only for **range** = 'A' or 'I' and **iu** – **il** = **n** – 1.

6: **info** – INTEGER

info = 0 unless the function detects an error (see Section 6).

6 Error Indicators and Warnings

info = $-i$

If **info** = $-i$, parameter i had an illegal value on entry. The parameters are numbered as follows:

1: **jobz**, 2: **range**, 3: **uplo**, 4: **n**, 5: **a**, 6: **lda**, 7: **vl**, 8: **vu**, 9: **il**, 10: **iu**, 11: **abstol**, 12: **m**, 13: **w**, 14: **z**, 15: **ldz**, 16: **isuppz**, 17: **work**, 18: **lwork**, 19: **iwork**, 20: **liwork**, 21: **info**.

It is possible that **info** refers to a parameter that is omitted from the MATLAB interface. This usually indicates that an error in one of the other input parameters has caused an incorrect value to be inferred.

info > 0

nag_lapack_dsyevr (f08fd) failed to converge.

7 Accuracy

The computed eigenvalues and eigenvectors are exact for a nearby matrix $(A + E)$, where

$$\|E\|_2 = O(\epsilon)\|A\|_2,$$

and ϵ is the *machine precision*. See Section 4.7 of Anderson *et al.* (1999) for further details.

8 Further Comments

The total number of floating-point operations is proportional to n^3 .

The complex analogue of this function is nag_lapack_zheevr (f08fr).

9 Example

This example finds the eigenvalues with indices in the range [2, 3], and the corresponding eigenvectors, of the symmetric matrix

$$A = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 2 & 3 & 4 \\ 3 & 3 & 3 & 4 \\ 4 & 4 & 4 & 4 \end{pmatrix}.$$

Information on required and provided workspace is also output.

9.1 Program Text

```

function f08fd_example

fprintf('f08fd example results\n\n');

% Find eigenvalues 2:3 of A
a = [1, 2, 3, 4;
     0, 2, 3, 4;
     0, 0, 3, 4;
     0, 0, 0, 4];

jobz = 'Vectors';
range = 'I';
uplo = 'Upper';
vl = 0;
vu = 0;
il = nag_int(2);
iu = nag_int(3);
abstol = 0;

[~, m, w, z, ~, info] = ...
    f08fd(...
        jobz, range, uplo, a, vl, vu, il, iu, abstol);

fprintf(' Eigenvalues numbered %d to %d are:\n    ', il, iu);
fprintf(' %7.4f', w(1:m));
fprintf('\n\n');

[ifail] = x04ca( ...
    'General', ' ', z, 'Selected eigenvectors');

```

9.2 Program Results

```

f08fd example results

Eigenvalues numbered 2 to 3 are:
-0.5146 -0.2943

Selected eigenvectors
      1      2
1 -0.5144  0.2767
2  0.4851 -0.6634
3  0.5420  0.6504
4 -0.4543 -0.2457

```
