

## NAG Toolbox

### **nag\_lapack\_zhpev (f08gn)**

## 1 Purpose

`nag_lapack_zhpev (f08gn)` computes all the eigenvalues and, optionally, all the eigenvectors of a complex  $n$  by  $n$  Hermitian matrix  $A$  in packed storage.

## 2 Syntax

```
[ap, w, z, info] = nag_lapack_zhpev(jobz, uplo, n, ap)
[ap, w, z, info] = f08gn(jobz, uplo, n, ap)
```

## 3 Description

The Hermitian matrix  $A$  is first reduced to real tridiagonal form, using unitary similarity transformations, and then the  $QR$  algorithm is applied to the tridiagonal matrix to compute the eigenvalues and (optionally) the eigenvectors.

## 4 References

Anderson E, Bai Z, Bischof C, Blackford S, Demmel J, Dongarra J J, Du Croz J J, Greenbaum A, Hammarling S, McKenney A and Sorensen D (1999) *LAPACK Users' Guide* (3rd Edition) SIAM, Philadelphia <http://www.netlib.org/lapack/lug>

Golub G H and Van Loan C F (1996) *Matrix Computations* (3rd Edition) Johns Hopkins University Press, Baltimore

## 5 Parameters

### 5.1 Compulsory Input Parameters

1: **jobz** – CHARACTER(1)

Indicates whether eigenvectors are computed.

**jobz** = 'N'

Only eigenvalues are computed.

**jobz** = 'V'

Eigenvalues and eigenvectors are computed.

*Constraint:* **jobz** = 'N' or 'V'.

2: **uplo** – CHARACTER(1)

If **uplo** = 'U', the upper triangular part of  $A$  is stored.

If **uplo** = 'L', the lower triangular part of  $A$  is stored.

*Constraint:* **uplo** = 'U' or 'L'.

3: **n** – INTEGER

$n$ , the order of the matrix  $A$ .

*Constraint:* **n**  $\geq 0$ .

4: **ap(:)** – COMPLEX (KIND=nag\_wp) array

The dimension of the array **ap** must be at least  $\max(1, \mathbf{n} \times (\mathbf{n} + 1)/2)$

The upper or lower triangle of the  $n$  by  $n$  Hermitian matrix  $A$ , packed by columns.

More precisely,

if **uplo** = 'U', the upper triangle of  $A$  must be stored with element  $A_{ij}$  in  $\mathbf{ap}(i + j(j - 1)/2)$  for  $i \leq j$ ;

if **uplo** = 'L', the lower triangle of  $A$  must be stored with element  $A_{ij}$  in  $\mathbf{ap}(i + (2n - j)(j - 1)/2)$  for  $i \geq j$ .

## 5.2 Optional Input Parameters

None.

## 5.3 Output Parameters

1: **ap(:)** – COMPLEX (KIND=nag\_wp) array

The dimension of the array **ap** will be  $\max(1, \mathbf{n} \times (\mathbf{n} + 1)/2)$

**ap** stores the values generated during the reduction to tridiagonal form. The elements of the diagonal and the off-diagonal of the tridiagonal matrix overwrite the corresponding elements of  $A$ .

2: **w(n)** – REAL (KIND=nag\_wp) array

The eigenvalues in ascending order.

3: **z(ldz,:)** – COMPLEX (KIND=nag\_wp) array

The first dimension, **ldz**, of the array **z** will be

if **jobz** = 'V', **ldz** =  $\max(1, \mathbf{n})$ ;  
otherwise **ldz** = 1.

The second dimension of the array **z** will be  $\max(1, \mathbf{n})$  if **jobz** = 'V' and 1 otherwise.

If **jobz** = 'V', **z** contains the orthonormal eigenvectors of the matrix  $A$ , with the  $i$ th column of  $Z$  holding the eigenvector associated with **w(i)**.

If **jobz** = 'N', **z** is not referenced.

4: **info** – INTEGER

**info** = 0 unless the function detects an error (see Section 6).

## 6 Error Indicators and Warnings

**info** =  $-i$

If **info** =  $-i$ , parameter  $i$  had an illegal value on entry. The parameters are numbered as follows:

1: **jobz**, 2: **uplo**, 3: **n**, 4: **ap**, 5: **w**, 6: **z**, 7: **ldz**, 8: **work**, 9: **rwork**, 10: **info**.

It is possible that **info** refers to a parameter that is omitted from the MATLAB interface. This usually indicates that an error in one of the other input parameters has caused an incorrect value to be inferred.

**info** > 0

If **info** =  $i$ , the algorithm failed to converge;  $i$  off-diagonal elements of an intermediate tridiagonal form did not converge to zero.

## 7 Accuracy

The computed eigenvalues and eigenvectors are exact for a nearby matrix  $(A + E)$ , where

$$\|E\|_2 = O(\epsilon)\|A\|_2,$$

and  $\epsilon$  is the *machine precision*. See Section 4.7 of Anderson *et al.* (1999) for further details.

## 8 Further Comments

Each eigenvector is normalized so that the element of largest absolute value is real.

The total number of floating-point operations is proportional to  $n^3$ .

The real analogue of this function is nag\_lapack\_dspev (f08ga).

## 9 Example

This example finds all the eigenvalues of the Hermitian matrix

$$A = \begin{pmatrix} 1 & 2-i & 3-i & 4-i \\ 2+i & 2 & 3-2i & 4-2i \\ 3+i & 3+2i & 3 & 4-3i \\ 4+i & 4+2i & 4+3i & 4 \end{pmatrix},$$

together with approximate error bounds for the computed eigenvalues.

### 9.1 Program Text

```
function f08gn_example

fprintf('f08gn example results\n\n');

% A is Hermitian matrix stored in symmetric (Upper) packed format
uplo = 'U';
n = nag_int(4);
ap = [1 + 0i;
      2 - 1i;      2 + 0i;
      3 - 1i;      3 - 2i;      3 + 0i;
      4 - 1i;      4 - 2i;      4 - 3i;      4 + 0i];

% Eigenvalues of A only
jobz = 'No vectors';
[apf, w, ~, info] = f08gn( ...
    jobz, uplo, n, ap);

disp('Eigenvalues');
disp(w');

% Eigenvalue error bound
errbnd = x02aj*max(abs(w(1)),abs(w(end)));
disp('Error estimate for the eigenvalues');
fprintf('%11.1e\n',errbnd);
```

### 9.2 Program Results

```
f08gn example results

Eigenvalues
-4.2443   -0.6886     1.1412    13.7916

Error estimate for the eigenvalues
1.5e-15
```

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