NAG Toolbox

nag_lapack_dtrsna (f08ql)

1 Purpose

nag_lapack_dtrsna (f08ql) estimates condition numbers for specified eigenvalues and/or right eigenvectors of a real upper quasi-triangular matrix.

2 Syntax

```
[s, sep, m, info] = nag_lapack_dtrsna(job, howmny, select, t, vl, vr, mm, 'n', n)
[s, sep, m, info] = f08ql(job, howmny, select, t, vl, vr, mm, 'n', n)
```

3 Description

nag_lapack_dtrsna (f08ql) estimates condition numbers for specified eigenvalues and/or right eigenvectors of a real upper quasi-triangular matrix T in canonical Schur form. These are the same as the condition numbers of the eigenvalues and right eigenvectors of an original matrix $A = ZTZ^{\mathrm{T}}$ (with orthogonal Z), from which T may have been derived.

nag_lapack_dtrsna (f08ql) computes the reciprocal of the condition number of an eigenvalue λ_i as

$$s_i = \frac{|v^{\mathrm{H}}u|}{\|u\|_E\|v\|_E},$$

where u and v are the right and left eigenvectors of T, respectively, corresponding to λ_i. This reciprocal condition number always lies between zero (i.e., ill-conditioned) and one (i.e., well-conditioned).

An approximate error estimate for a computed eigenvalue λ_i is then given by

$$\frac{\epsilon\|T\|}{s_i},$$

where ϵ is the ***machine precision***.

To estimate the reciprocal of the condition number of the right eigenvector corresponding to λ_i, the function first calls nag_lapack_dtrexc (f08qf) to reorder the eigenvalues so that λ_i is in the leading position:

$$T = Q\begin{pmatrix} \lambda_i & c^{\mathrm{T}} \\ 0 & T_{22} \end{pmatrix}Q^{\mathrm{T}}.$$

The reciprocal condition number of the eigenvector is then estimated as sep_i, the smallest singular value of the matrix $(T_{22} - \lambda_i I)$. This number ranges from zero (i.e., ill-conditioned) to very large (i.e., well-conditioned).

An approximate error estimate for a computed right eigenvector u corresponding to λ_i is then given by

$$\frac{\epsilon\|T\|}{sep_i}.$$

4 References

Golub G H and Van Loan C F (1996) *Matrix Computations* (3rd Edition) Johns Hopkins University Press, Baltimore

5 Parameters

5.1 Compulsory Input Parameters

1: **job** – CHARACTER(1)

Indicates whether condition numbers are required for eigenvalues and/or eigenvectors.

job = 'E'
Condition numbers for eigenvalues only are computed.

job = 'V'
Condition numbers for eigenvectors only are computed.

job = 'B'
Condition numbers for both eigenvalues and eigenvectors are computed.

Constraint: **job** = 'E', 'V' or 'B'.

2: **howmny** – CHARACTER(1)

Indicates how many condition numbers are to be computed.

howmny = 'A'
Condition numbers for all eigenpairs are computed.

howmny = 'S'
Condition numbers for selected eigenpairs (as specified by **select**) are computed.

Constraint: **howmny** = 'A' or 'S'.

3: **select**(:) – LOGICAL array

The dimension of the array **select** must be at least $\max(1, \mathbf{n})$ if **howmny** = 'S', and at least 1 otherwise

Specifies the eigenpairs for which condition numbers are to be computed if **howmny** = 'S'. To select condition numbers for the eigenpair corresponding to the real eigenvalue λ_j, **select**(j) must be set *true*. To select condition numbers corresponding to a complex conjugate pair of eigenvalues λ_j and λ_{j+1}, **select**(j) and/or **select**$(j+1)$ must be set to *true*.

If **howmny** = 'A', **select** is not referenced.

4: **t**$(ldt, :)$ – REAL (KIND=nag_wp) array

The first dimension of the array **t** must be at least $\max(1, \mathbf{n})$.

The second dimension of the array **t** must be at least $\max(1, \mathbf{n})$.

The n by n upper quasi-triangular matrix T in canonical Schur form, as returned by nag_lapack_dhseqr (f08pe).

5: **vl**$(ldvl, :)$ – REAL (KIND=nag_wp) array

The first dimension, $ldvl$, of the array **vl** must satisfy

if **job** = 'E' or 'B', $ldvl \geq \max(1, \mathbf{n})$;
if **job** = 'V', $ldvl \geq 1$.

The second dimension of the array **vl** must be at least $\max(1, \mathbf{mm})$ if **job** = 'E' or 'B' and at least 1 if **job** = 'V'.

If **job** = 'E' or 'B', **vl** must contain the left eigenvectors of T (or of any matrix QTQ^{T} with Q orthogonal) corresponding to the eigenpairs specified by **howmny** and **select**. The eigenvectors **must** be stored in consecutive columns of **vl**, as returned by nag_lapack_dhsein (f08pk) or nag_lapack_dtrevc (f08qk).

If **job** = 'V', **vl** is not referenced.

6: **vr**$(ldvr,:)$ – REAL (KIND=nag_wp) array

The first dimension, $ldvr$, of the array **vr** must satisfy

if **job** = 'E' or 'B', $ldvr \geq \max(1, \mathbf{n})$;
if **job** = 'V', $ldvr \geq 1$.

The second dimension of the array **vr** must be at least $\max(1, \mathbf{mm})$ if **job** = 'E' or 'B' and at least 1 if **job** = 'V'.

If **job** = 'E' or 'B', **vr** must contain the right eigenvectors of T (or of any matrix QTQ^{T} with Q orthogonal) corresponding to the eigenpairs specified by **howmny** and **select**. The eigenvectors **must** be stored in consecutive columns of **vr**, as returned by nag_lapack_dhsein (f08pk) or nag_lapack_dtrevc (f08qk).

If **job** = 'V', **vr** is not referenced.

7: **mm** – INTEGER

The number of elements in the arrays **s** and **sep**, and the number of columns in the arrays **vl** and **vr** (if used). The precise number required, m, is n if **howmny** = 'A'; if **howmny** = 'S', m is obtained by counting 1 for each selected real eigenvalue, and 2 for each selected complex conjugate pair of eigenvalues (see **select**), in which case $0 \leq m \leq n$.

Constraint: **mm** $\geq$ **m**.

5.2 Optional Input Parameters

1: **n** – INTEGER

Default: the first dimension of the array **t** and the second dimension of the array **t**. (An error is raised if these dimensions are not equal.)

n, the order of the matrix T.

Constraint: **n** ≥ 0.

5.3 Output Parameters

1: **s**(:) – REAL (KIND=nag_wp) array

The dimension of the array **s** will be $\max(1, \mathbf{mm})$ if **job** = 'E' or 'B' and 1 otherwise

The reciprocal condition numbers of the selected eigenvalues if **job** = 'E' or 'B', stored in consecutive elements of the array. Thus **s**(j), **sep**(j) and the jth columns of **vl** and **vr** all correspond to the same eigenpair (but not in general the jth eigenpair unless all eigenpairs have been selected). For a complex conjugate pair of eigenvalues, two consecutive elements of **s** are set to the same value.

If **job** = 'V', **s** is not referenced.

2: **sep**(:) – REAL (KIND=nag_wp) array

The dimension of the array **sep** will be $\max(1, \mathbf{mm})$ if **job** = 'V' or 'B' and 1 otherwise

The estimated reciprocal condition numbers of the selected right eigenvectors if **job** = 'V' or 'B', stored in consecutive elements of the array. For a complex eigenvector, two consecutive elements of **sep** are set to the same value. If the eigenvalues cannot be reordered to compute **sep**(j), then **sep**(j) is set to zero; this can only occur when the true value would be very small anyway.

If **job** = 'E', **sep** is not referenced.

3: **m** – INTEGER

m, the number of elements of **s** and/or **sep** actually used to store the estimated condition numbers. If **howmny** = 'A', **m** is set to n.

4: **info** – INTEGER

info $= 0$ unless the function detects an error (see Section 6).

6 Error Indicators and Warnings

info $= -i$

If **info** $= -i$, parameter i had an illegal value on entry. The parameters are numbered as follows:

1: **job**, 2: **howmny**, 3: **select**, 4: **n**, 5: **t**, 6: **ldt**, 7: **vl**, 8: **ldvl**, 9: **vr**, 10: **ldvr**, 11: **s**, 12: **sep**, 13: **mm**, 14: **m**, 15: **work**, 16: **ldwork**, 17: **iwork**, 18: **info**.

It is possible that **info** refers to a parameter that is omitted from the MATLAB interface. This usually indicates that an error in one of the other input parameters has caused an incorrect value to be inferred.

7 Accuracy

The computed values sep_i may over estimate the true value, but seldom by a factor of more than 3.

8 Further Comments

For a description of canonical Schur form, see the document for nag_lapack_dhseqr (f08pe).

The complex analogue of this function is nag_lapack_ztrsna (f08qy).

9 Example

This example computes approximate error estimates for all the eigenvalues and right eigenvectors of the matrix T, where

$$T = \begin{pmatrix} 0.7995 & -0.1144 & 0.0060 & 0.0336 \\ 0.0000 & -0.0994 & 0.2478 & 0.3474 \\ 0.0000 & -0.6483 & -0.0994 & 0.2026 \\ 0.0000 & 0.0000 & 0.0000 & -0.1007 \end{pmatrix}.$$

9.1 Program Text

```
function f08ql_example

fprintf('f08ql example results\n\n');

% Schur form matrix T
n = nag_int(4);
t = [0.7995, -0.1144,  0.0060,  0.0336;
     0,      -0.0994,  0.2478,  0.3474;
     0,      -0.6483, -0.0994,  0.2026;
     0,       0,       0,      -0.1007];

% Calculate left and right eigenvectors of T
job = 'Both';
howmny = 'All';
select = [false];
[select, vl, vr, m, info] = ...
f08qk( ...
       job, howmny, select, t, zeros(n,n), zeros(n,n), n);

% Estimate condition numbers of eigenvalues and right eigenvectors
[s, sep, m, info] = f08ql( ...
    job, howmny, select, t, vl, vr, m);

disp('s');
disp(s');
disp('sep');
```

```
disp(sep');
tnorm = norm(t,1);
disp('Approximate error estimates for eigenvalues of T (machine-dependent)');
fprintf('%11.1e',x02aj*tnorm./s);
fprintf('\n\n%s %s\n', 'Approximate error estimates for right', ...
        'eigenvectors (machine-dependent)');
fprintf('%11.1e',x02aj*tnorm./sep);
fprintf('\n');
```

9.2 Program Results

```
     f08ql example results

s
    0.9937    0.7028    0.7028    0.5711

sep
    0.6252    0.3743    0.3743    0.3125

Approximate error estimates for eigenvalues of T (machine-dependent)
    9.6e-17    1.4e-16    1.4e-16    1.7e-16

Approximate error estimates for right eigenvectors (machine-dependent)
    1.5e-16    2.6e-16    2.6e-16    3.1e-16
```