NAG Toolbox

nag_lapack_dsbgv (f08ua)

1 Purpose

nag_lapack_dsbgv (f08ua) computes all the eigenvalues and, optionally, the eigenvectors of a real generalized symmetric-definite banded eigenproblem, of the form

$$Az = \lambda Bz,$$

where A and B are symmetric and banded, and B is also positive definite.

2 Syntax

```
[ab, bb, w, z, info] = nag_lapack_dsbgv(jobz, uplo, ka, kb, ab, bb, 'n', n)
[ab, bb, w, z, info] = f08ua(jobz, uplo, ka, kb, ab, bb, 'n', n)
```

3 Description

The generalized symmetric-definite band problem

$$Az = \lambda Bz$$

is first reduced to a standard band symmetric problem

$$Cx = \lambda x,$$

where C is a symmetric band matrix, using Wilkinson's modification to Crawford's algorithm (see Crawford (1973) and Wilkinson (1977)). The symmetric eigenvalue problem is then solved for the eigenvalues and the eigenvectors, if required, which are then backtransformed to the eigenvectors of the original problem.

The eigenvectors are normalized so that the matrix of eigenvectors, Z, satisfies

$$Z^{\mathrm{T}}AZ = \Lambda \quad \text{and} \quad Z^{\mathrm{T}}BZ = I,$$

where Λ is the diagonal matrix whose diagonal elements are the eigenvalues.

4 References

Anderson E, Bai Z, Bischof C, Blackford S, Demmel J, Dongarra J J, Du Croz J J, Greenbaum A, Hammarling S, McKenney A and Sorensen D (1999) *LAPACK Users' Guide* (3rd Edition) SIAM, Philadelphia http://www.netlib.org/lapack/lug

Crawford C R (1973) Reduction of a band-symmetric generalized eigenvalue problem *Comm. ACM* **16** 41–44

Golub G H and Van Loan C F (1996) *Matrix Computations* (3rd Edition) Johns Hopkins University Press, Baltimore

Wilkinson J H (1977) Some recent advances in numerical linear algebra *The State of the Art in Numerical Analysis* (ed D A H Jacobs) Academic Press

5 Parameters

5.1 Compulsory Input Parameters

1: **jobz** – CHARACTER(1)

Indicates whether eigenvectors are computed.

jobz = 'N'
Only eigenvalues are computed.

jobz = 'V'
Eigenvalues and eigenvectors are computed.

Constraint: **jobz** = 'N' or 'V'.

2: **uplo** – CHARACTER(1)

If **uplo** = 'U', the upper triangles of A and B are stored.

If **uplo** = 'L', the lower triangles of A and B are stored.

Constraint: **uplo** = 'U' or 'L'.

3: **ka** – INTEGER

If **uplo** = 'U', the number of superdiagonals, k_a, of the matrix A.

If **uplo** = 'L', the number of subdiagonals, k_a, of the matrix A.

Constraint: **ka** ≥ 0.

4: **kb** – INTEGER

If **uplo** = 'U', the number of superdiagonals, k_b, of the matrix B.

If **uplo** = 'L', the number of subdiagonals, k_b, of the matrix B.

Constraint: **ka** $\geq$ **kb** ≥ 0.

5: **ab**$(ldab,:)$ – REAL (KIND=nag_wp) array

The first dimension of the array **ab** must be at least **ka** $+ 1$.

The second dimension of the array **ab** must be at least $\max(1,\mathbf{n})$.

The upper or lower triangle of the n by n symmetric band matrix A.

The matrix is stored in rows 1 to $k_a + 1$, more precisely,

if **uplo** = 'U', the elements of the upper triangle of A within the band must be stored with element A_{ij} in $\mathbf{ab}(k_a + 1 + i - j, j)$ for $\max(1, j - k_a) \leq i \leq j$;

if **uplo** = 'L', the elements of the lower triangle of A within the band must be stored with element A_{ij} in $\mathbf{ab}(1 + i - j, j)$ for $j \leq i \leq \min(n, j + k_a)$.

6: **bb**$(ldbb,:)$ – REAL (KIND=nag_wp) array

The first dimension of the array **bb** must be at least **kb** $+ 1$.

The second dimension of the array **bb** must be at least $\max(1,\mathbf{n})$.

The upper or lower triangle of the n by n symmetric band matrix B.

The matrix is stored in rows 1 to $k_b + 1$, more precisely,

if **uplo** = 'U', the elements of the upper triangle of B within the band must be stored with element B_{ij} in $\mathbf{bb}(k_b + 1 + i - j, j)$ for $\max(1, j - k_b) \leq i \leq j$;

if **uplo** = 'L', the elements of the lower triangle of B within the band must be stored with element B_{ij} in $\mathbf{bb}(1 + i - j, j)$ for $j \leq i \leq \min(n, j + k_b)$.

5.2 Optional Input Parameters

1: **n** – INTEGER

Default: the second dimension of the arrays **ab**, **bb**. (An error is raised if these dimensions are not equal.)

n, the order of the matrices A and B.

Constraint: $\mathbf{n} \geq 0$.

5.3 Output Parameters

1: **ab**$(ldab,:)$ – REAL (KIND=nag_wp) array

The first dimension of the array **ab** will be $\mathbf{ka} + 1$.

The second dimension of the array **ab** will be $\max(1, \mathbf{n})$.

The contents of **ab** are overwritten.

2: **bb**$(ldbb,:)$ – REAL (KIND=nag_wp) array

The first dimension of the array **bb** will be $\mathbf{kb} + 1$.

The second dimension of the array **bb** will be $\max(1, \mathbf{n})$.

The factor S from the split Cholesky factorization $B = S^{\mathrm{T}}S$, as returned by nag_lapack_dpbstf (f08uf).

3: **w**$(\mathbf{n})$ – REAL (KIND=nag_wp) array

The eigenvalues in ascending order.

4: **z**$(ldz,:)$ – REAL (KIND=nag_wp) array

The first dimension, ldz, of the array **z** will be

if **jobz** = 'V', $ldz = \max(1, \mathbf{n})$;
otherwise $ldz = 1$.

The second dimension of the array **z** will be $\max(1, \mathbf{n})$ if **jobz** = 'V' and 1 otherwise.

If **jobz** = 'V', **z** contains the matrix Z of eigenvectors, with the ith column of Z holding the eigenvector associated with $\mathbf{w}(i)$. The eigenvectors are normalized so that $Z^{\mathrm{T}}BZ = I$.

If **jobz** = 'N', **z** is not referenced.

5: **info** – INTEGER

info = 0 unless the function detects an error (see Section 6).

6 Error Indicators and Warnings

info $= -i$

If **info** $= -i$, parameter i had an illegal value on entry. The parameters are numbered as follows:

1: **jobz**, 2: **uplo**, 3: **n**, 4: **ka**, 5: **kb**, 6: **ab**, 7: **ldab**, 8: **bb**, 9: **ldbb**, 10: **w**, 11: **z**, 12: **ldz**, 13: **work**, 14: **info**.

It is possible that **info** refers to a parameter that is omitted from the MATLAB interface. This usually indicates that an error in one of the other input parameters has caused an incorrect value to be inferred.

info > 0

If **info** $= i$ and $i \leq$ **n**, the algorithm failed to converge; i off-diagonal elements of an intermediate tridiagonal form did not converge to zero.

If **info** $= i$ and $i >$ **n**, if **info** $=$ **n** $+ i$, for $1 \leq i \leq$ **n**, then nag_lapack_dpbstf (f08uf) returned **info** $= i$: B is not positive definite. The factorization of B could not be completed and no eigenvalues or eigenvectors were computed.

7 Accuracy

If B is ill-conditioned with respect to inversion, then the error bounds for the computed eigenvalues and vectors may be large, although when the diagonal elements of B differ widely in magnitude the eigenvalues and eigenvectors may be less sensitive than the condition of B would suggest. See Section 4.10 of Anderson *et al.* (1999) for details of the error bounds.

8 Further Comments

The total number of floating-point operations is proportional to n^3 if **jobz** = 'V' and, assuming that $n \gg k_a$, is approximately proportional to $n^2 k_a$ otherwise.

The complex analogue of this function is nag_lapack_zhbgv (f08un).

9 Example

This example finds all the eigenvalues of the generalized band symmetric eigenproblem $Az = \lambda Bz$, where

$$A = \begin{pmatrix} 0.24 & 0.39 & 0.42 & 0 \\ 0.39 & -0.11 & 0.79 & 0.63 \\ 0.42 & 0.79 & -0.25 & 0.48 \\ 0 & 0.63 & 0.48 & -0.03 \end{pmatrix} \quad \text{and} \quad B = \begin{pmatrix} 2.07 & 0.95 & 0 & 0 \\ 0.95 & 1.69 & -0.29 & 0 \\ 0 & -0.29 & 0.65 & -0.33 \\ 0 & 0 & -0.33 & 1.17 \end{pmatrix}.$$

9.1 Program Text

```
function f08ua_example

fprintf('f08ua example results\n\n');

% Symmetric banded matrices A and B stored in symmetric banded format
uplo = 'U';
ka = nag_int(2);
ab = [0,     0,     0.42,  0.63;
      0,     0.39,  0.79,  0.48;
      0.24, -0.11, -0.25, -0.03];
kb = nag_int(1);
bb = [0,     0.95, -0.29, -0.33;
      2.07,  1.69,  0.65,  1.17];

% Eigenvalues only of Ax = lambda Bx
jobz = 'No vectors';
[~, ~, w, ~, info] = f08ua( ...
     jobz, uplo, ka, kb, ab, bb);

disp('Eigenvalues');
disp(w');
```

9.2 Program Results

```
f08ua example results

Eigenvalues
   -0.8305   -0.6401    0.0992    1.8525
```