NAG Toolbox

nag_lapack_dsbgst (f08ue)

1 Purpose

nag_lapack_dsbgst (f08ue) reduces a real symmetric-definite generalized eigenproblem $Az = \lambda Bz$ to the standard form $Cy = \lambda y$, where A and B are band matrices, A is a real symmetric matrix, and B has been factorized by nag_lapack_dpbstf (f08uf).

2 Syntax

```
[ab, x, info] = nag_lapack_dsbgst(vect, uplo, ka, kb, ab, bb, 'n', n)
[ab, x, info] = f08ue(vect, uplo, ka, kb, ab, bb, 'n', n)
```

3 Description

To reduce the real symmetric-definite generalized eigenproblem $Az = \lambda Bz$ to the standard form $Cy = \lambda y$, where A, B and C are banded, nag_lapack_dsbgst (f08ue) must be preceded by a call to nag_lapack_dpbstf (f08uf) which computes the split Cholesky factorization of the positive definite matrix B: $B = S^{\mathrm{T}}S$. The split Cholesky factorization, compared with the ordinary Cholesky factorization, allows the work to be approximately halved.

This function overwrites A with $C = X^{\mathrm{T}}AX$, where $X = S^{-1}Q$ and Q is a orthogonal matrix chosen (implicitly) to preserve the bandwidth of A. The function also has an option to allow the accumulation of X, and then, if z is an eigenvector of C, Xz is an eigenvector of the original system.

4 References

Crawford C R (1973) Reduction of a band-symmetric generalized eigenvalue problem *Comm. ACM* **16** 41–44

Kaufman L (1984) Banded eigenvalue solvers on vector machines *ACM Trans. Math. Software* **10** 73–86

5 Parameters

5.1 Compulsory Input Parameters

1: **vect** – CHARACTER(1)

Indicates whether X is to be returned.

vect = 'N'
: X is not returned.

vect = 'V'
: X is returned.

Constraint: **vect** = 'N' or 'V'.

2: **uplo** – CHARACTER(1)

Indicates whether the upper or lower triangular part of A is stored.

uplo = 'U'
: The upper triangular part of A is stored.

uplo = 'L'
The lower triangular part of A is stored.

Constraint: **uplo** = 'U' or 'L'.

3: **ka** – INTEGER

If **uplo** = 'U', the number of superdiagonals, k_a, of the matrix A.

If **uplo** = 'L', the number of subdiagonals, k_a, of the matrix A.

Constraint: **ka** ≥ 0.

4: **kb** – INTEGER

If **uplo** = 'U', the number of superdiagonals, k_b, of the matrix B.

If **uplo** = 'L', the number of subdiagonals, k_b, of the matrix B.

Constraint: **ka** $\geq$ **kb** ≥ 0.

5: **ab**$(ldab,:)$ – REAL (KIND=nag_wp) array

The first dimension of the array **ab** must be at least **ka** $+ 1$.

The second dimension of the array **ab** must be at least $\max(1, \mathbf{n})$.

The upper or lower triangle of the n by n symmetric band matrix A.

The matrix is stored in rows 1 to $k_a + 1$, more precisely,

if **uplo** = 'U', the elements of the upper triangle of A within the band must be stored with element A_{ij} in **ab**$(k_a + 1 + i - j, j)$ for $\max(1, j - k_a) \leq i \leq j$;

if **uplo** = 'L', the elements of the lower triangle of A within the band must be stored with element A_{ij} in **ab**$(1 + i - j, j)$ for $j \leq i \leq \min(n, j + k_a)$.

6: **bb**$(ldbb,:)$ – REAL (KIND=nag_wp) array

The first dimension of the array **bb** must be at least **kb** $+ 1$.

The second dimension of the array **bb** must be at least $\max(1, \mathbf{n})$.

The banded split Cholesky factor of B as specified by **uplo**, **n** and **kb** and returned by nag_lapack_dpbstf (f08uf).

5.2 Optional Input Parameters

1: **n** – INTEGER

Default: the second dimension of the arrays **ab**, **bb**.

n, the order of the matrices A and B.

Constraint: **n** ≥ 0.

5.3 Output Parameters

1: **ab**$(ldab,:)$ – REAL (KIND=nag_wp) array

The first dimension of the array **ab** will be **ka** $+ 1$.

The second dimension of the array **ab** will be $\max(1, \mathbf{n})$.

The upper or lower triangle of **ab** stores the corresponding upper or lower triangle of C as specified by **uplo**.

2: **x**(ldx, :) – REAL (KIND=nag_wp) array

The first dimension, ldx, of the array **x** will be

if **vect** = 'V', $ldx = \max(1, \mathbf{n})$;
if **vect** = 'N', $ldx = 1$.

The second dimension of the array **x** will be $\max(1, \mathbf{n})$ if **vect** = 'V' and at least 1 if **vect** = 'N'.

The n by n matrix $X = S^{-1}Q$, if **vect** = 'V'.

If **vect** = 'N', **x** is not referenced.

3: **info** – INTEGER

info $= 0$ unless the function detects an error (see Section 6).

6 Error Indicators and Warnings

info $= -i$

If **info** $= -i$, parameter i had an illegal value on entry. The parameters are numbered as follows:

1: **vect**, 2: **uplo**, 3: **n**, 4: **ka**, 5: **kb**, 6: **ab**, 7: **ldab**, 8: **bb**, 9: **ldbb**, 10: **x**, 11: **ldx**, 12: **work**, 13: **info**.

It is possible that **info** refers to a parameter that is omitted from the MATLAB interface. This usually indicates that an error in one of the other input parameters has caused an incorrect value to be inferred.

7 Accuracy

Forming the reduced matrix C is a stable procedure. However it involves implicit multiplication by B^{-1}. When nag_lapack_dsbgst (f08ue) is used as a step in the computation of eigenvalues and eigenvectors of the original problem, there may be a significant loss of accuracy if B is ill-conditioned with respect to inversion.

8 Further Comments

The total number of floating-point operations is approximately $6n^2k_B$, when **vect** = 'N', assuming $n \gg k_A, k_B$; there are an additional $(3/2)n^3(k_B/k_A)$ operations when **vect** = 'V'.

The complex analogue of this function is nag_lapack_zhbgst (f08us).

9 Example

This example computes all the eigenvalues of $Az = \lambda Bz$, where

$$A = \begin{pmatrix} 0.24 & 0.39 & 0.42 & 0.00 \\ 0.39 & -0.11 & 0.79 & 0.63 \\ 0.42 & 0.79 & -0.25 & 0.48 \\ 0.00 & 0.63 & 0.48 & -0.03 \end{pmatrix} \quad \text{and} \quad B = \begin{pmatrix} 2.07 & 0.95 & 0.00 & 0.00 \\ 0.95 & 1.69 & -0.29 & 0.00 \\ 0.00 & -0.29 & 0.65 & -0.33 \\ 0.00 & 0.00 & -0.33 & 1.17 \end{pmatrix}.$$

Here A is symmetric, B is symmetric positive definite, and A and B are treated as band matrices. B must first be factorized by nag_lapack_dpbstf (f08uf). The program calls nag_lapack_dsbgst (f08ue) to reduce the problem to the standard form $Cy = \lambda y$, then nag_lapack_dsbtrd (f08he) to reduce C to tridiagonal form, and nag_lapack_dsterf (f08jf) to compute the eigenvalues.

9.1 Program Text

```
     function f08ue_example

fprintf('f08ue example results\n\n');

% Sove Az = lambda Bz
% A and B are the symmetric banded positive definite matrices:
n = 4;
% A has 2 off-diagonals
ka = nag_int(2);
a = [ 0.24,  0.39,  0.42,  0.00;
      0.39, -0.11,  0.79,  0.63;
      0.42,  0.79, -0.25,  0.48;
      0.00,  0.63,  0.48, -0.03];
% B has 1 off-diagonal
kb = nag_int(1);
b = [ 2.07   0.95   0.00   0.00;
      0.95   1.69  -0.29   0.00;
      0.00  -0.29   0.65  -0.33;
      0.00   0.00  -0.33   1.17];

% Convert to general banded format ...
[~, ab, ifail] = f01zc( ...
 'P', ka, ka, a, zeros(ka+ka+1,n));
[~, bb, ifail] = f01zc( ...
 'P', kb, kb, b, zeros(kb+kb+1,n));
% ... and chop to give 'Upper' symmetric banded format
ab = ab(1:ka+1,1:n);
bb = bb(1:kb+1,1:n);

% Factorize B
uplo = 'Upper';
[ub, info] = f08uf( ...
     uplo, kb, bb);

% Reduce problem to standard form Cy = lambda*y
vect = 'N';
[cb, x, info] = f08ue( ...
        vect, uplo, ka, kb, ab, ub);

% Find eigenvalues lambda
jobz = 'No Vectors';
[~, w, ~, info] = f08ha( ...
  jobz, uplo, ka, cb);

disp('Eigenvalues:');
disp(w');
```

9.2 Program Results

```
     f08ue example results

Eigenvalues:
   -0.8305   -0.6401    0.0992    1.8525
```