NAG Toolbox

nag_blast_dgb_norm (f16rb)

1 Purpose

nag_blast_dgb_norm (f16rb) calculates the value of the 1-norm, the ∞-norm, the Frobenius norm or the maximum absolute value of the elements of a real m by n band matrix stored in banded form.

It can also be used to compute the value of the 2-norm of a row n-vector or a column m-vector.

2 Syntax

```
[result] = nag_blast_dgb_norm(job, m, kl, ku, ab, 'n', n)
[result] = f16rb(job, m, kl, ku, ab, 'n', n)
```

3 Description

Given a real m by n banded matrix, A, nag_blast_dgb_norm (f16rb) calculates one of the values given by

$$\|A\|_1 = \max_j \sum_{i=1}^{m} |a_{ij}| \qquad \text{(the 1-norm of } A\text{)},$$

$$\|A\|_\infty = \max_i \sum_{j=1}^{n} |a_{ij}| \qquad \text{(the } \infty\text{-norm of } A\text{)},$$

$$\|A\|_F = \left(\sum_{i=1}^{m} \sum_{j=1}^{n} |a_{ij}|^2 \right)^{1/2} \qquad \text{(the Frobenius norm of } A\text{)}, \quad \text{or}$$

$$\max_{i,j} |a_{ij}| \qquad \text{(the maximum absolute element value of } A\text{)}.$$

If m or n is 1 then additionally nag_blast_dgb_norm (f16rb) can calculate the value $\|A\|_2 = \sqrt{\sum a_i^2}$ (the 2-norm of A).

4 References

Basic Linear Algebra Subprograms Technical (BLAST) Forum (2001) *Basic Linear Algebra Subprograms Technical (BLAST) Forum Standard* University of Tennessee, Knoxville, Tennessee http://www.netlib.org/blas/blast-forum/blas-report.pdf

5 Parameters

5.1 Compulsory Input Parameters

1: **job** – CHARACTER(1)

Specifies the value to be returned.

job = 'O'
The 1-norm.

job = 'T'
The 2-norm of a row or column vector.

job = 'I'
The ∞-norm.

job = 'F'
The Frobenius (or Euclidean) norm.

job = 'M'
The value $\max_{i,j} |a_{ij}|$ (not a norm).

Constraints:

job = 'O', 'T', 'I', 'F' or 'M';
if **job** = 'T', **m** = 1 or **n** = 1.

2: **m** – INTEGER

m, the number of rows of the matrix A. If **m** ≤ 0 on input, nag_blast_dgb_norm (f16rb) returns 0.

3: **kl** – INTEGER

k_l, the number of subdiagonals within the band of A. If **kl** ≤ 0 on input, nag_blast_dgb_norm (f16rb) returns 0.

4: **ku** – INTEGER

k_u, the number of superdiagonals within the band of A. If **ku** ≤ 0 on input, nag_blast_dgb_norm (f16rb) returns 0.

5: **ab**$(ldab, :)$ – REAL (KIND=nag_wp) array

The first dimension of the array **ab** must be at least **kl** + **ku** + 1.

The second dimension of the array **ab** must be at least max$(1, \mathbf{n})$.

The m by n band matrix A.

The matrix is stored in rows 1 to $k_l + k_u + 1$, more precisely, the element A_{ij} must be stored in

$$\mathbf{ab}(k_u + 1 + i - j, j) \quad \text{for } \max(1, j - k_u) \leq i \leq \min(m, j + k_l).$$

5.2 Optional Input Parameters

1: **n** – INTEGER

Default: the second dimension of the array **ab**.

n, the number of columns of the matrix A. If **n** ≤ 0 on input, nag_blast_dgb_norm (f16rb) returns 0.

5.3 Output Parameters

1: **result**

The result of the function.

6 Error Indicators and Warnings

If any constraint on an input parameter is violated, an error message is printed and program execution is terminated.

7 Accuracy

The BLAS standard requires accurate implementations which avoid unnecessary over/underflow (see Section 2.7 of Basic Linear Algebra Subprograms Technical (BLAST) Forum (2001)).

8 Further Comments

None.

9 Example

Calculates the various norms of a 6 by 4 banded matrix with two subdiagonals and one superdiagonal.

9.1 Program Text

```
     function f16rb_example

fprintf('f16rb example results\n\n');

kl = nag_int(2);
ku = nag_int(2);
m  = nag_int(6);
a = [ 1, 1, 0, 0;
      2, 2, 2, 0;
      3, 3, 3, 3;
      0, 4, 4, 4;
      0, 0, 5, 5;
      0, 0, 0, 6];
ab = zeros(5, 6);
% Convert a to packed storage
[a, ab, ifail] = f01zc( ...
 'p', kl, ku, a, ab);

fprintf('\nNorms of banded matrix ab:\n\n');

r_one = f16rb('o', m, kl, ku, ab);
fprintf('One norm           = %9.4f\n', r_one);

r_inf = f16rb('i', m, kl, ku, ab);
fprintf('Infinity norm      = %9.4f\n', r_inf);

r_fro = f16rb('f', m, kl, ku, ab);
fprintf('Frobenious norm    = %9.4f\n', r_fro);

r_max = f16rb('m', m, kl, ku, ab);
fprintf('Maximum norm       = %9.4f\n', r_max);
```

9.2 Program Results

```
     f16rb example results

Norms of banded matrix ab:

One norm           =   18.0000
Infinity norm      =   12.0000
Frobenious norm    =   13.5647
Maximum norm       =    6.0000
```