NAG Toolbox

nag_tsa_uni_arima_prelim (g13ad)

1   Purpose

nag_tsa_uni_arima_prelim (g13ad) calculates preliminary estimates of the parameters of an autoregressive integrated moving average (ARIMA) model from the autocorrelation function of the appropriately differenced times series.

2   Syntax

```
[par, rv, isf, ifail] = nag_tsa_uni_arima_prelim(mr, r, xv, npar, 'nk', nk)

[par, rv, isf, ifail] = g13ad(mr, r, xv, npar, 'nk', nk)
```

3   Description

Preliminary estimates of the p non-seasonal autoregressive parameters $\phi_1, \phi_2, \ldots, \phi_p$ and the q non-seasonal moving average parameters $\theta_1, \theta_2, \ldots, \theta_q$ may be obtained from the sample autocorrelations relating to lags 1 to $p+q$, i.e., $r_1, \ldots, r_{p+q}$, of the differenced $\nabla^d \nabla_s^D x_t$, where x_t is assumed to follow a (possibly) seasonal ARIMA model (see Section 3 in nag_tsa_uni_arima_estim (g13ae) for the specification of an ARIMA model).

Taking $r_0 = 1$ and $r_{-k} = r_k$, the ϕ_i, for $i = 1, 2, \ldots, p$ are the solutions to the equations

$$r_{q+i-1}\phi_1 + r_{q+i-2}\phi_2 + \cdots + r_{q+i-p}\phi_p = r_{q+i}, \quad i = 1, 2, \ldots, p.$$

The θ_j, for $j = 1, 2, \ldots, q$, are obtained from the solutions to the equations

$$c_j = \tau_0\tau_j + \tau_1\tau_{j+1} + \cdots + \tau_{q+j}\tau_q, \quad j = 0, 1, \ldots, q$$

(Cramer Wold-factorization), by setting

$$\theta_j = -\frac{\tau_j}{\tau_0},$$

where c_j are the 'covariances' modified in a two stage process by the autoregressive parameters.

Stage 1:

$$\begin{aligned} d_j &= r_j - \phi_1 r_{j-1} - \cdots - \phi_p r_{j-p}, & j = 0, 1, \ldots, q; \\ d_j &= 0, & j = q+1, q+2, \ldots, p+q. \end{aligned}$$

Stage 2:

$$c_j = d_j - \phi_1 d_{j+1} - \phi_2 d_{j+2} - \cdots - \phi_p d_{j+p}, \quad j = 0, 1, \ldots, q.$$

The P seasonal autoregressive parameters $\Phi_1, \Phi_2, \ldots, \Phi_P$ and the Q seasonal moving average parameters $\Theta_1, \Theta_2, \ldots, \Theta_Q$ are estimated in the same way as the non-seasonal parameters, but each r_j is replaced in the calculation by $r_{s \times j}$, where s is the seasonal period.

An estimate of the residual variance is obtained by successively reducing the sample variance, first for non-seasonal, and then for seasonal, parameter estimates. If moving average parameters are estimated, the variance is reduced by a multiplying factor of τ_0^2, but otherwise by c_0.

4   References

Box G E P and Jenkins G M (1976) *Time Series Analysis: Forecasting and Control* (Revised Edition) Holden–Day

5   Parameters

5.1   Compulsory Input Parameters

1:     **mr**(7) – INTEGER array

The orders vector (p, d, q, P, D, Q, s) of the ARIMA model whose parameters are to be estimated. p, q, P and Q refer respectively to the number of autoregressive (ϕ), moving average (θ), seasonal autoregressive (Φ) and seasonal moving average (Θ) parameters. d, D and s refer respectively to the order of non-seasonal differencing, the order of seasonal differencing and the seasonal period.

Constraints:

$p, d, q, P, D, Q, s \geq 0$;
$p + q + P + Q > 0$;
$s \neq 1$;
if $s = 0$, $P + D + Q = 0$;
if $s > 1$, $P + D + Q > 0$.

2:     **r**(**nk**) – REAL (KIND=nag_wp) array

The autocorrelations (starting at lag 1), which must have been calculated after the time series has been appropriately differenced.

Constraint: $-1.0 \leq \mathbf{r}(i) \leq 1.0$, for $i = 1, 2, \ldots, \mathbf{nk}$.

3:     **xv** – REAL (KIND=nag_wp)

The series sample variance, calculated after appropriate differencing has been applied to the series.

Constraint: $\mathbf{xv} > 0.0$.

4:     **npar** – INTEGER

The exact number of parameters specified in the model by array **mr**.

Constraint: $\mathbf{npar} = p + q + P + Q$.

5.2   Optional Input Parameters

1:     **nk** – INTEGER

Default: the dimension of the array **r**.

The maximum lag of the autocorrelations in array **r**.

Constraint: $\mathbf{nk} \geq \max(p + q, s \times (P + Q))$.

5.3   Output Parameters

1:     **par**(**npar**) – REAL (KIND=nag_wp) array

The first **npar** elements of **par** contain the preliminary estimates of the ARIMA model parameters, in standard order.

2:     **rv** – REAL (KIND=nag_wp)

An estimate of the residual variance of the preliminarily estimated model.

3:     **isf**(4) – INTEGER array

Contains success/failure indicators, one for each of the four types of parameter (autoregressive, moving average, seasonal autoregressive, seasonal moving average).

The indicator has the interpretation:

0   No parameter of this type is in the model.

1   Parameters of this type appear in the model and satisfactory preliminary estimates of this type were obtained.

−1   Parameters of this type appear in the model but satisfactory preliminary estimates of this type were not obtainable. The estimates of this type of parameter were set to 0.0 in array **par**.

4:   **ifail** – INTEGER

ifail = 0 unless the function detects an error (see Section 5).

6   Error Indicators and Warnings

Errors or warnings detected by the function:

ifail = 1

On entry, the orders vector **mr** is invalid. One of the constraints in Section 5 has been violated.

ifail = 2

On entry, $\mathbf{nk} < \max(p + q, s \times (P + Q))$. There are not enough autocorrelations to enable the required model to be estimated.

ifail = 3

On entry, at least one element of **r** lies outside the range $[-1.0, 1.0]$.

ifail = 4

On entry, $\mathbf{xv} \le 0.0$.

ifail = 5

On entry, $\mathbf{npar} \ne p + q + P + Q$.

ifail = 6

On entry, the workspace array wa is too small. See Section 5 for the minimum size formula.

ifail = 7 (*warning*)

Satisfactory parameter estimates could not be obtained for all parameter types in the model. Inspect array **isf** for indicators of the parameter type(s) which could not be estimated.

ifail = −99

An unexpected error has been triggered by this routine. Please contact NAG.

ifail = −399

Your licence key may have expired or may not have been installed correctly.

ifail = −999

Dynamic memory allocation failed.

7   Accuracy

The performance of the algorithm is conditioned by the roots of the autoregressive and moving average operators. If these are not close to unity in modulus, the errors, e, should satisfy $e < 100\epsilon$ where ϵ is *machine precision*.

8   Further Comments

The time taken by nag_tsa_uni_arima_prelim (g13ad) is approximately proportional to $\left(p^3 + q^2 + P^3 + Q^2\right)$.

9   Example

This example reads the sample autocorrelations to lag 40 and the sample variance of the lagged and doubly differenced series of airline passenger totals (Box and Jenkins example series G (see Box and Jenkins (1976))). Preliminary estimates of the parameters of the $(0, 1, 1, 0, 1, 1, 12)$ model are obtained by a call to nag_tsa_uni_arima_prelim (g13ad).

9.1   Program Text

```
    function g13ad_example

fprintf('g13ad example results\n\n');

% Data
r = [-0.32804;     0.09850;   -0.21854;     0.05585;     0.04679;
      0.04135;    -0.07989;    0.00335;     0.13973;    -0.04022;
      0.07618;    -0.40583;    0.18239;    -0.05057;     0.16094;
     -0.15900;     0.09152;   -0.03474;     0.05195;    -0.14417;
      0.04264;    -0.08170;    0.23389;    -0.02828;    -0.09001;
      0.03050;    -0.02046;    0.05522;    -0.02048;    -0.06651;
     -0.02940;     0.20204;   -0.13953;     0.10098;    -0.20849;
      0.03338;     0.00829;    0.07082;    -0.04457;    -0.01216];

% orders
mr = [nag_int(0);1;1;0;1;1;12];
% variance
xv = 0.00213;

npar = nag_int(2);

% Calculate preliminary estimates
[par, rv, isf, ifail] = g13ad( ...
                               mr, r, xv, npar);

% Display results
fprintf('Parameter estimation success/failure indicator');
fprintf('%4d', isf);
fprintf('\n\nARIMA model parameter values ');
fprintf('%10.5f', par);
fprintf('\n\nResidual variance %10.5f\n', rv);
```

9.2   Program Results

```
    g13ad example results

Parameter estimation success/failure indicator   0   1   0   1

ARIMA model parameter values    0.37390   0.51237

Residual variance    0.00148
```