NAG Toolbox

nag_tsa_multi_corrmat_cross (g13dm)

1 Purpose

nag_tsa_multi_corrmat_cross (g13dm) calculates the sample cross-correlation (or cross-covariance) matrices of a multivariate time series.

2 Syntax

```
[wmean, r0, r, ifail] = nag_tsa_multi_corrmat_cross(matrix, k, m, w, 'n', n)
[wmean, r0, r, ifail] = g13dm(matrix, k, m, w, 'n', n)
```

3 Description

Let $W_t = (w_{1t}, w_{2t}, \dots, w_{kt})^{\mathrm{T}}$, for $t = 1, 2, \dots, n$, denote n observations of a vector of k time series. The sample cross-covariance matrix at lag l is defined to be the k by k matrix $\hat{C}(l)$, whose (i, j)th element is given by

$$\hat{C}_{ij}(l) = \frac{1}{n} \sum_{t=l+1}^{n} \left(w_{i(t-l)} - \bar{w}_i\right)\left(w_{jt} - \bar{w}_j\right), \quad l = 0, 1, 2, \dots, m, \ i = 1, 2, \dots, k \text{ and } j = 1, 2, \dots, k,$$

where $\bar{w}_i$ and $\bar{w}_j$ denote the sample means for the ith and jth series respectively. The sample cross-correlation matrix at lag l is defined to be the k by k matrix $\hat{R}(l)$, whose (i, j)th element is given by

$$\hat{R}_{ij}(l) = \frac{\hat{C}_{ij}(l)}{\sqrt{\hat{C}_{ii}(0)\hat{C}_{jj}(0)}}, \quad l = 0, 1, 2, \dots, m, \ i = 1, 2, \dots, k \text{ and } j = 1, 2, \dots, k.$$

The number of lags, m, is usually taken to be at most $n/4$.

If W_t follows a vector moving average model of order q, then it can be shown that the theoretical cross-correlation matrices $(R(l))$ are zero beyond lag q. In order to help spot a possible cut-off point, the elements of $\hat{R}(l)$ are usually compared to their approximate standard error of $1/\sqrt{n}$. For further details see, for example, Wei (1990).

The function uses a single pass through the data to compute the means and the cross-covariance matrix at lag zero. The cross-covariance matrices at further lags are then computed on a second pass through the data.

4 References

Wei W W S (1990) *Time Series Analysis: Univariate and Multivariate Methods* Addison–Wesley

West D H D (1979) Updating mean and variance estimates: An improved method *Comm. ACM* **22** 532–555

5 Parameters

5.1 Compulsory Input Parameters

1: **matrix** – CHARACTER(1)

Indicates whether the cross-covariance or cross-correlation matrices are to be computed.

matrix = 'V'
The cross-covariance matrices are computed.

matrix = 'R'
The cross-correlation matrices are computed.

Constraint: **matrix** = 'V' or 'R'.

2: **k** – INTEGER

k, the dimension of the multivariate time series.

Constraint: **k** ≥ 1.

3: **m** – INTEGER

m, the number of cross-correlation (or cross-covariance) matrices to be computed. If in doubt set **m** $= 10$. However it should be noted that **m** is usually taken to be at most **n**$/4$.

Constraint: $1 \leq$ **m** $<$ **n**.

4: **w**($kmax$, **n**) – REAL (KIND=nag_wp) array

$kmax$, the first dimension of the array, must satisfy the constraint $kmax \geq$ **k**.

w(i, t) must contain the observation w_{it}, for $i = 1, 2, \ldots, k$ and $t = 1, 2, \ldots, n$.

5.2 Optional Input Parameters

1: **n** – INTEGER

Default: the second dimension of the array **w**.

n, the number of observations in the series.

Constraint: **n** ≥ 2.

5.3 Output Parameters

1: **wmean**(**k**) – REAL (KIND=nag_wp) array

The means, $\bar{w}_i$, for $i = 1, 2, \ldots, k$.

2: **r0**($kmax$, **k**) – REAL (KIND=nag_wp) array

$kmax =$ **k**.

If $i \neq j$, then **r0**(i, j) contains an estimate of the (i, j)th element of the cross-correlation (or cross-covariance) matrix at lag zero, $\hat{R}_{ij}(0)$; if $i = j$, then if **matrix** = 'V', **r0**(i, i) contains the variance of the ith series, $\hat{C}_{ii}(0)$, and if **matrix** = 'R', **r0**(i, i) contains the standard deviation of the ith series, $\sqrt{\hat{C}_{ii}(0)}$.

If **ifail** $= 2$ and **matrix** = 'R', then on exit all the elements in **r0** whose computation involves the zero variance are set to zero.

3: **r**($kmax$, $kmax$, **m**) – REAL (KIND=nag_wp) array

$kmax =$ **k**.

r(i, j, l) contains an estimate of the (i, j)th element of the cross-correlation (or cross-covariance) at lag l, $\hat{R}_{ij}(l)$, for $l = 1, 2, \ldots, m$, $i = 1, 2, \ldots, k$ and $j = 1, 2, \ldots, k$.

If **ifail** $= 2$ and **matrix** = 'R', then on exit all the elements in **r** whose computation involves the zero variance are set to zero.

4: **ifail** – INTEGER

ifail $= 0$ unless the function detects an error (see Section 5).

6 Error Indicators and Warnings

Errors or warnings detected by the function:

ifail $= 1$

On entry, **matrix** $\neq$ 'V' or 'R',
or **k** < 1,
or **n** < 2,
or **m** < 1,
or **m** $\geq$ **n**,
or $kmax <$ **k**.

ifail $= 2$ (*warning*)

On entry, at least one of the k series is such that all its elements are practically equal giving zero (or near zero) variance. In this case if **matrix** = 'R' all the correlations in **r0** and **r** involving this variance are set to zero.

ifail $= -99$

An unexpected error has been triggered by this routine. Please contact NAG.

ifail $= -399$

Your licence key may have expired or may not have been installed correctly.

ifail $= -999$

Dynamic memory allocation failed.

7 Accuracy

For a discussion of the accuracy of the one-pass algorithm used to compute the sample cross-covariances at lag zero see West (1979). For the other lags a two-pass algorithm is used to compute the cross-covariances; the accuracy of this algorithm is also discussed in West (1979). The accuracy of the cross-correlations will depend on the accuracy of the computed cross-covariances.

8 Further Comments

The time taken is roughly proportional to mnk^2.

9 Example

This program computes the sample cross-correlation matrices of two time series of length 48, up to lag 10. It also prints the cross-correlation matrices together with plots of symbols indicating which elements of the correlation matrices are significant. Three * represent significance at the 0.5% level, two * represent significance at the 1% level and a single * represents significance at the 5% level. The * are plotted above or below the line depending on whether the elements are significant in the positive or negative direction.

9.1 Program Text

```
function g13dm_example

fprintf('g13dm example results\n\n');

w = [-1.49, -1.62, 5.20, 6.23, 6.21, 5.86, 4.09, 3.18, 2.62, 1.49, 1.17, ...
      0.85, -0.35, 0.24, 2.44, 2.58, 2.04, 0.40, 2.26, 3.34, 5.09, 5.00, ...
      4.78,  4.11, 3.45, 1.65, 1.29, 4.09, 6.32, 7.50, 3.89, 1.58, 5.21, ...
      5.25,  4.93, 7.38, 5.87, 5.81, 9.68, 9.07, 7.29, 7.84, 7.55, 7.32, ...
      7.97,  7.76, 7.00, 8.35;
```

```
      7.34,  6.35, 6.96, 8.54, 6.62, 4.97, 4.55, 4.81, 4.75, 4.76,10.88, ...
     10.01, 11.62,10.36, 6.40, 6.24, 7.93, 4.04, 3.73, 5.60, 5.35, 6.81, ...
      8.27,  7.68, 6.65, 6.08,10.25, 9.14,17.75,13.30, 9.63, 6.80, 4.08, ...
      5.06,  4.94, 6.65, 7.94,10.76,11.89, 5.85, 9.01, 7.50,10.02,10.38, ...
      8.15, 8.37, 10.73, 12.14];
[k,n] = size(w);
k     = nag_int(k);
matrix = 'R';
m = nag_int(10);
[wmean, r0, r, ifail] = g13dm( ...
        matrix, k, m, w);

disp('The means');
disp(wmean');
disp('Cross-Correlation Matrices');
disp('Lag = 0');
disp(r0);
for l = 1:m
  fprintf('Lag = %d\n',l);
  disp(r(:,:,l));
end
sn1 = 1/sqrt(n);
fprintf('Standard error = 1/sqrt(n) = %7.4f\n\n',sn1);

disp('Tables Of Indicator Symbols');
fprintf('\nFor Lags 1 to %d\n',m);
lhs = {'             0.005  :'; '        +     0.01   :';
        '             0.05   :';
        '   Sig. Level        :- - - - - - - - - -  Lags';
        '             0.05   :';
        '        -     0.01   :'; '             0.005  :'};
c = sn1*[3.29, 2.58, 1.96, 0, -1.96, -2.58, -3.29];
for i = 1:k
  for j=1:k
    if i==j
      fprintf('\nAuto-correlation function for series %d\n', i);
    else
      fprintf('\nCross-correlation function for series %d and series %d\n', ...
       i, j);
    end
    rhs = lhs;
    for t = 1:m
      for u = 1:3
 if r(i,j,t)>c(u)
   rhs{u} = strcat(rhs{u},'*');
 end
      end
      for u = 5:7
 if r(i,j,t)<c(u)
   rhs{u} = strcat(rhs{u},'*');
 end
      end
    end
    fprintf('\n');
    fprintf('%s\n',rhs{1:end});
  end
end
```

9.2 Program Results

```
     g13dm example results

The means
    4.3702    7.8675

Cross-Correlation Matrices
Lag = 0
    2.8176    0.2493
    0.2493    2.8149
```

```
Lag = 1
    0.7359    0.1743
    0.2114    0.5546

Lag = 2
    0.4557    0.0764
    0.0693    0.2605

Lag = 3
    0.3792    0.0138
    0.0260   -0.0381

Lag = 4
    0.3224    0.1100
    0.0933   -0.2359

Lag = 5
    0.3411    0.2694
    0.0872   -0.2501

Lag = 6
    0.3631    0.3436
    0.1323   -0.2265

Lag = 7
    0.2800    0.4254
    0.2069   -0.1285

Lag = 8
    0.2480    0.5217
    0.1970   -0.0846

Lag = 9
    0.2398    0.2664
    0.2537    0.0745

Lag = 10
    0.1619   -0.0197
    0.2667    0.0047

Standard error = 1/sqrt(n) =  0.1443

Tables Of Indicator Symbols

For Lags 1 to 10

Auto-correlation function for series 1

              0.005  :*
        +     0.01   :***
              0.05   :******
   Sig. Level        :- - - - - - - - - -  Lags
              0.05   :
        -     0.01   :
              0.005  :

Cross-correlation function for series 1 and series 2

              0.005  :*
        +     0.01   :**
              0.05   :***
   Sig. Level        :- - - - - - - - - -  Lags
              0.05   :
        -     0.01   :
              0.005  :

Cross-correlation function for series 2 and series 1

              0.005  :
        +     0.01   :
              0.05   :
```

```
   Sig. Level        :- - - - - - - - - - -  Lags
              0.05   :
        -     0.01   :
              0.005  :

Auto-correlation function for series 2

              0.005  :*
        +     0.01   :*
              0.05   :*
   Sig. Level        :- - - - - - - - - - -  Lags
              0.05   :
        -     0.01   :
              0.005  :
```