NAG Toolbox

nag_tsa_cp_binary_user (g13ne)

1 Purpose

nag_tsa_cp_binary_user (g13ne) detects change points in a univariate time series, that is, the time points at which some feature of the data, for example the mean, changes. Change points are detected using binary segmentation for a user-supplied cost function.

2 Syntax

```
[tau, user, ifail] = nag_tsa_cp_binary_user(n, beta, chgpfn, 'minss', minss,
'mdepth', mdepth, 'user', user)
```

```
[tau, user, ifail] = g13ne(n, beta, chgpfn, 'minss', minss, 'mdepth', mdepth,
'user', user)
```

3 Description

Let $y_{1:n} = \{y_j : j = 1, 2, \ldots, n\}$ denote a series of data and $\tau = \{\tau_i : i = 1, 2, \ldots, m\}$ denote a set of m ordered (strictly monotonic increasing) indices known as change points with $1 \leq \tau_i \leq n$ and $\tau_m = n$. For ease of notation we also define $\tau_0 = 0$. The m change points, τ, split the data into m segments, with the ith segment being of length n_i and containing $y_{\tau_{i-1}+1:\tau_i}$.

Given a cost function, $C(y_{\tau_{i-1}+1:\tau_i})$, nag_tsa_cp_binary_user (g13ne) gives an approximate solution to

$$\underset{m,\tau}{\text{minimize}} \sum_{i=1}^{m} \left(C(y_{\tau_{i-1}+1:\tau_i}) + \beta \right)$$

where β is a penalty term used to control the number of change points. The solution is obtained in an iterative manner as follows:

1. Set $u = 1$, $w = n$ and $k = 0$
2. Set $k = k + 1$. If $k > K$, where K is a user-supplied control parameter, then terminate the process for this segment.
3. Find v that minimizes
$$C(y_{u:v}) + C(y_{v+1:w})$$
4. Test
$$C(y_{u:v}) + C(y_{v+1:w}) + \beta < C(y_{u:w}) \qquad (1)$$
5. If inequality (1) is false then the process is terminated for this segment.
6. If inequality (1) is true, then v is added to the set of change points, and the segment is split into two subsegments, $y_{u:v}$ and $y_{v+1:w}$. The whole process is repeated from step 2 independently on each subsegment, with the relevant changes to the definition of u and w (i.e., w is set to v when processing the left hand subsegment and u is set to $v + 1$ when processing the right hand subsegment.

The change points are ordered to give τ.

4 References

Chen J and Gupta A K (2010) *Parametric Statistical Change Point Analysis With Applications to Genetics Medicine and Finance* **Second Edition** Birkhl̀user

5 Parameters

5.1 Compulsory Input Parameters

1: **n** – INTEGER

n, the length of the time series.

Constraint: $\mathbf{n} \geq 2$.

2: **beta** – REAL (KIND=nag_wp)

β, the penalty term.

There are a number of standard ways of setting β, including:

SIC or BIC
: $\beta = p \times \log(n)$.

AIC
: $\beta = 2p$.

Hannan-Quinn
: $\beta = 2p \times \log(\log(n))$.

where p is the number of parameters being treated as estimated in each segment. The value of p will depend on the cost function being used.

If no penalty is required then set $\beta = 0$. Generally, the smaller the value of β the larger the number of suggested change points.

3: **chgpfn** – SUBROUTINE, supplied by the user.

chgpfn must calculate a proposed change point, and the associated costs, within a specified segment.

```
[v, cost, user, info] = chgpfn(side, u, w, minss, user, info)
```

Input Parameters

1: **side** – INTEGER

Flag indicating what **chgpfn** must calculate and at which point of the Binary Segmentation it has been called.

side $= -1$
: only $C(y_{u:w})$ need be calculated and returned in **cost**(1), neither **v** nor the other elements of **cost** need be set. In this case, $u = 1$ and $w =$ n.

side $= 0$
: all elements of **cost** and **v** must be set. In this case, $u = 1$ and $w =$ n.

side $= 1$
: the segment, $y_{u:w}$, is a left hand side subsegment from a previous iteration of the Binary Segmentation algorithm. All elements of **cost** and **v** must be set.

side $= 2$
: the segment, $y_{u:w}$, is a right hand side subsegment from a previous iteration of the Binary Segmentation algorithm. All elements of **cost** and **v** must be set.

The distinction between **side** $= 1$ and 2 may allow for **chgpfn** to be implemented in a more efficient manner. See section Section 10 for one such example.

The first call to **chgpfn** will always have **side** $= -1$ and the second call will always have **side** $= 0$. All subsequent calls will be made with **side** $= 1$ or 2.

2: **u** – INTEGER

u, the start of the segment of interest.

3: **w** – INTEGER

w, the end of the segment of interest.

4: **minss** – INTEGER

The minimum distance between two change points, as passed to nag_tsa_cp_binary_user (g13ne).

5: **user** – INTEGER array

chgpfn is called from nag_tsa_cp_binary_user (g13ne) with the object supplied to nag_tsa_cp_binary_user (g13ne).

6: **info** – INTEGER

info $= 0$.

Output Parameters

1: **v** – INTEGER

If **side** $= -1$ then **v** need not be set.

if **side** $\neq -1$ then v, the proposed change point. That is, the value which minimizes

$$\underset{v}{\text{minimize}} C(y_{u:v}) + C(y_{v+1:w})$$

for $v = u +$ **minss** $- 1$ to $w -$ **minss**.

2: **cost**(**3**) – REAL (KIND=nag_wp) array

Costs associated with the proposed change point, v.

If **side** $= -1$ then **cost**$(1) = C(y_{u:w})$ and the remaining two elements of **cost** need not be set.

If **side** $\neq -1$ then

cost$(1) = C(y_{u:v}) + C(y_{v+1:w})$.

cost$(2) = C(y_{u:v})$.

cost$(3) = C(y_{v+1:w})$.

3: **user** – INTEGER array

4: **info** – INTEGER

In most circumstances **info** should remain unchanged.

If **info** is set to a strictly positive value then nag_tsa_cp_binary_user (g13ne) terminates with **ifail** $= 51$.

If **info** is set to a strictly negative value the current segment is skipped (i.e., no change points are considered in this segment) and nag_tsa_cp_binary_user (g13ne) continues as normal. If **info** was set to a strictly negative value at any point and no other errors occur then nag_tsa_cp_binary_user (g13ne) will terminate with **ifail** $= 52$.

5.2 Optional Input Parameters

1: **minss** – INTEGER

Default: 2

The minimum distance between two change points, that is $\tau_i - \tau_{i-1} \geq$ **minss**.

Constraint: **minss** ≥ 2.

2: **mdepth** – INTEGER

Default: 0

K, the maximum depth for the iterative process, which in turn puts an upper limit on the number of change points with $m \leq 2^K$.

If $K \leq 0$ then no limit is put on the depth of the iterative process and no upper limit is put on the number of change points.

3: **user** – INTEGER array

user is not used by nag_tsa_cp_binary_user (g13ne), but is passed to **chgpfn**. Note that for large objects it may be more efficient to use a global variable which is accessible from the m-files than to use **user**.

5.3 Output Parameters

1: **tau**$(ntau)$ – INTEGER array

The dimension of the array **tau** will be $ntau$

The location of the change points. The ith segment is defined by $y_{(\tau_{i-1}+1)}$ to y_{τ_i}, where $\tau_0 = 0$ and $\tau_i =$ **tau**$(i), 1 \leq i \leq m$.

2: **user** – INTEGER array

3: **ifail** – INTEGER

ifail $= 0$ unless the function detects an error (see Section 5).

6 Error Indicators and Warnings

Errors or warnings detected by the function:

ifail $= 11$

Constraint: **n** ≥ 2.

ifail $= 31$

Constraint: **minss** ≥ 2.

ifail $= 51$

User requested termination by setting .

ifail $= 52$ (*warning*)

User requested a segment to be skipped by setting .

ifail $= -99$

An unexpected error has been triggered by this routine. Please contact NAG.

ifail $= -399$

Your licence key may have expired or may not have been installed correctly.

ifail $= -999$

Dynamic memory allocation failed.

7 Accuracy

Not applicable.

8 Further Comments

nag_tsa_cp_binary (g13nd) performs the same calculations for a cost function selected from a provided set of cost functions. If the required cost function belongs to this provided set then nag_tsa_cp_binary (g13nd) can be used without the need to provide a cost function routine.

9 Example

This example identifies changes in the scale parameter, under the assumption that the data has a gamma distribution, for a simulated dataset with 100 observations. A penalty, β of 3.6 is used and the minimum segment size is set to 3. The shape parameter is fixed at 2.1 across the whole input series.

The cost function used is

$$C(y_{\tau_{i-1}+1:\tau_i}) = 2an_i(\log S_i - \log (an_i))$$

where a is a shape parameter that is fixed for all segments and $n_i = \tau_i - \tau_{i-1} + 1$.

9.1 Program Text

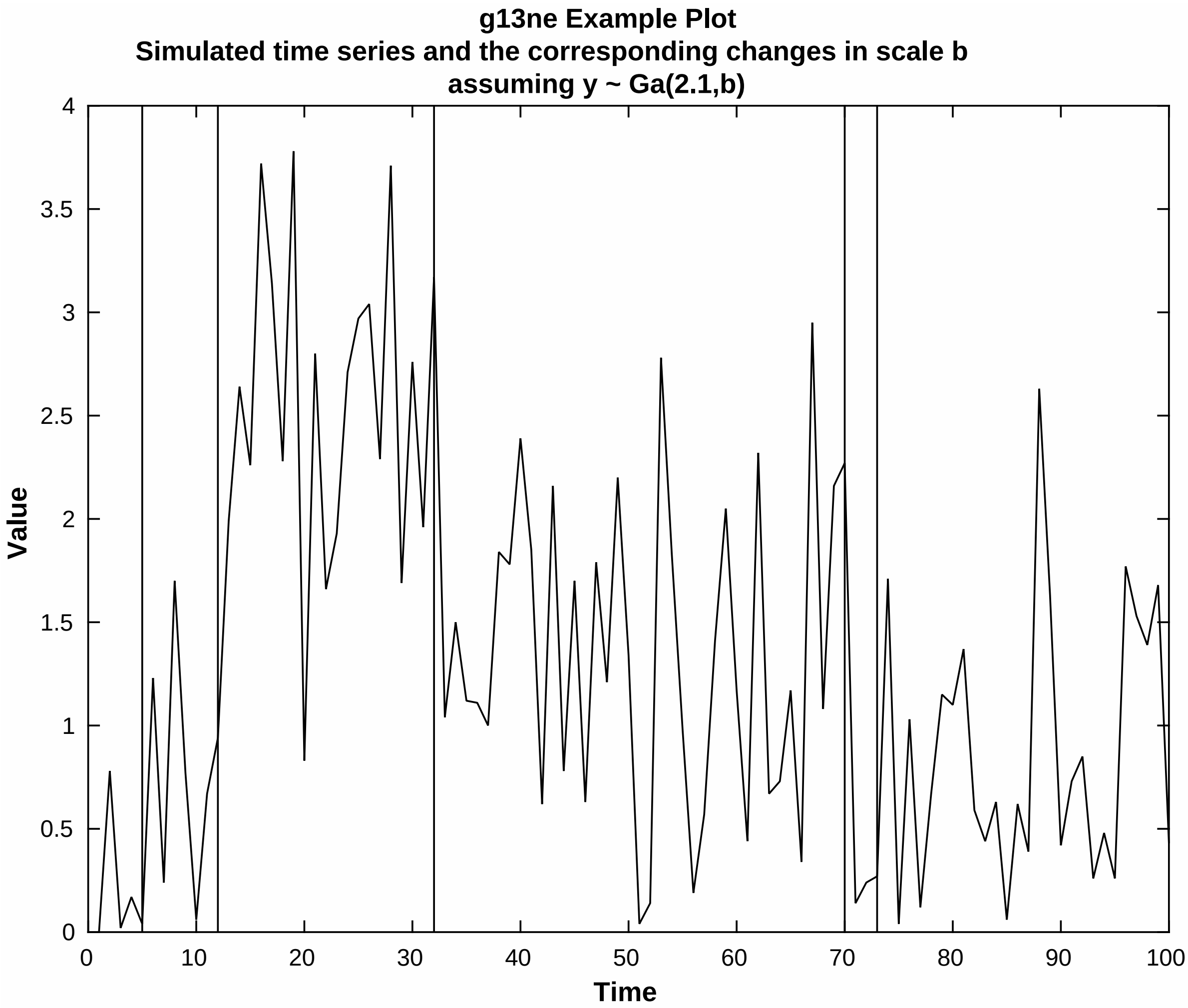
```
      function g13ne_example

fprintf('g13ne example results\n\n');

% Input series
y = [ 0.00; 0.78; 0.02; 0.17; 0.04; 1.23; 0.24; 1.70; 0.77; 0.06;
      0.67; 0.94; 1.99; 2.64; 2.26; 3.72; 3.14; 2.28; 3.78; 0.83;
      2.80; 1.66; 1.93; 2.71; 2.97; 3.04; 2.29; 3.71; 1.69; 2.76;
      1.96; 3.17; 1.04; 1.50; 1.12; 1.11; 1.00; 1.84; 1.78; 2.39;
      1.85; 0.62; 2.16; 0.78; 1.70; 0.63; 1.79; 1.21; 2.20; 1.34;
      0.04; 0.14; 2.78; 1.83; 0.98; 0.19; 0.57; 1.41; 2.05; 1.17;
      0.44; 2.32; 0.67; 0.73; 1.17; 0.34; 2.95; 1.08; 2.16; 2.27;
      0.14; 0.24; 0.27; 1.71; 0.04; 1.03; 0.12; 0.67; 1.15; 1.10;
      1.37; 0.59; 0.44; 0.63; 0.06; 0.62; 0.39; 2.63; 1.63; 0.42;
      0.73; 0.85; 0.26; 0.48; 0.26; 1.77; 1.53; 1.39; 1.68; 0.43];

% Shape parameter used in the cost function
a = 2.1;

% Length of the input series
n = nag_int(numel(y));

% Need some persisteny workspace in the user function
work = zeros(2*n,1);

% The input series, workspace and shape parameter
% constitute the information that needs to be passed to the
% costfun, so pack them together into a cell array which will
% get passed through the NAG function
user = {y; a; work};

% Penalty term
beta = 3.4;
```

```
% Drop small regions
minss = nag_int(3);

[tau] = g13ne(n,beta,@chgpfn,'minss',minss,'user',user);

% Print the results
fprintf('  -- Change Points --\n');
fprintf('  Number     Position\n');
fprintf(' ====================\n');
for i = 1:numel(tau)
  fprintf(' %4d       %6d\n', i, tau(i));
end

% Plot the results
fig1 = figure;

% Plot the original series
plot(y,'Color','red');

% Mark the change points, drop the last one as it is always
% at the end of the series
xpos = transpose(double(tau(1:end-1))*ones(1,2));
ypos = diag(ylim)*ones(2,numel(tau)-1);
line(xpos,ypos,'Color','black');

% Add labels and titles
title({'{\bf g13ne Example Plot}',
       'Simulated time series and the corresponding changes in scale b',
       'assuming y ~ Ga(2.1,b)'});
xlabel('{\bf Time}');
ylabel('{\bf Value}');

function [v,cost,user,info] = chgpfn(side,u,w,minss,user,info)
  % Function to calculate a proposed change point and associated cost
  % The cost is based on the likelihood of the gamma distribution
  y = user{1};
  a = user{2};
  work = user{3};

  % Calculate the first and last positions for potential change
  % points, conditional on the fact that each sub-segment must be
  % at least minss wide
  floc = u + minss - 1;
  lloc = w - minss;

  % In order to calculate the cost of having a change point at i, we
  % need to calculate C(y(floc:i)) and C(y(i+1:lloc)), where C(.) is
  % the cost function (based on the gamma distribution in this example).
  % Rather than calculate these values at each call to chgpfn we store
  % the values in work for later use

  % If side = 1 (i.e. we are working with a left hand sub-segment),
  % we already have C(y(floc:i)) for this value of floc, so only need
  % to calculate C(y(i+1:lloc)), similarly when side = 2 we only need
  % to calculate C(y(floc:i))
  % When side = -1, we need the cost of the full segment, which we do
  % in a forwards manner (calculating C(y(floc:i)) in the process), so
  % when side = 0 we only need to calculate C(y(i:1:lloc))

  % Get the intermediate costs
  ys = 0;
  dn = 0;
  if (side==0 | side==1)
    % work(2*i) = C(y(i+1:w))
    for i = w:-1:floc + 1
      dn = dn + 1;
      tmp = dn*a;
      ys = ys + y(i);
      work(2*i-2) = 2.0*tmp*(log(ys)-log(tmp));
    end
```

```
    % make sure we return the updated values of work
    user = {y; a; work};

  else
    % work(2*i-1) = C(y(u:i))
    if (side==-1)
      li = w;
    else
      li = lloc;
    end
    for i = u:li
      dn = dn + 1;
      tmp = dn*a;
      ys = ys + y(i);
      work(2*i-1) = 2.0*tmp*(log(ys)-log(tmp));
    end

    % make sure we return the updated values of work
    user = {y; a; work};
  end

  v = nag_int(0);
  cost = zeros(3,1);
  if (side>=0)
    % Need to find a potential change point
    v = nag_int(0);
    cost(1) = 0;

    % Loop over all possible change point locations
    for i = floc:lloc
      this_cost = work(2*i-1) + work(2*i);

      if (this_cost<cost(1) | v==0)
        % Update the proposed change point location
        v = nag_int(i);
        cost(1) = this_cost;
        cost(2) = work(2*i-1);
        cost(3) = work(2*i);
      end
    end

  else
    % Need to calculate the cost for the full segment
    cost(1) = work(2*w-1);

  % No need to populate the rest of COST or V
  end

  % Set info nonzero to terminate execution for any reason
  info = nag_int(0);
```

9.2 Program Results

```
     g13ne example results

  -- Change Points --
  Number     Position
 =====================
    1            5
    2           12
    3           32
    4           70
    5           73
    6          100
```



This example plot shows the original data series and the estimated change points.