## NAG Toolbox

## nag_mip_ilp_info (h02bz)

## 1 Purpose

nag_mip_ilp_info (h02bz) extracts more information associated with the solution of an integer programming problem computed by nag_mip_ilp_dense (h02bb).

## 2 Syntax

```
[bl, bu, clamda, istate, ifail] = nag_mip_ilp_info(n, m, iwork, rwork)
[bl, bu, clamda, istate, ifail] = h02bz(n, m, iwork, rwork)
```

Note: the interface to this routine has changed since earlier releases of the toolbox:
At Mark 22: liwork and lrwork were removed from the interface.

## 3 Description

nag_mip_ilp_info (h02bz) extracts the following information associated with the solution of an integer programming problem computed by nag_mip_ilp_dense (h02bb). The upper and lower bounds used for the solution, the Lagrange-multipliers (costs), and the status of the variables at the solution.

In the branch and bound method employed by nag_mip_ilp_dense (h02bb), the arrays bl and bu are used to impose restrictions on the values of the integer variables in each sub-problem. That is, if the variable $x_{j}$ is restricted to take value $v_{j}$ in a particular sub-problem, then $\mathbf{b l}(j)=\mathbf{b u}(j)=v_{j}$ is set in the sub-problem. Thus, on exit from this function, some of the elements of $\mathbf{b l}$ and $\mathbf{b u}$ which correspond to integer variables may contain these imposed values, rather than those originally supplied to nag_mip_ilp_dense (h02bb).

## 4 References

None.

## 5 Parameters

### 5.1 Compulsory Input Parameters

1: $\quad \mathbf{n}$ - INTEGER
This must be the same argument $\mathbf{n}$ as supplied to nag_mip_ilp_dense (h02bb).
Constraint: $\mathbf{n}>0$.
2: $\mathbf{m}$ - INTEGER
This must be the same argument $\mathbf{m}$ as supplied to nag_mip_ilp_dense (h02bb).
Constraint: $\mathbf{m} \geq 0$.
3: iwork(liwork) - INTEGER array
This must be the same argument iwork as supplied to nag_mip_ilp_dense (h02bb). It is used to pass information from nag_mip_ilp_dense (h02bb) to nag_mip_ilp_info (h02bz) and therefore the contents of this array must not be changed before calling nag_mip_ilp_info (h02bz).

4: $\quad$ rwork (lrwork) - REAL (KIND=nag_wp) array
This must be the same argument rwork as supplied to nag_mip_ilp_dense (h02bb). It is used to pass information from nag_mip_ilp_dense (h02bb) to nag_mip_ilp_info (h02bz) and therefore the contents of this array must not be changed before calling nag_mip_ilp_info (h02bz).

### 5.2 Optional Input Parameters

None.

### 5.3 Output Parameters

1: $\quad \mathbf{b l}(\mathbf{n}+\mathbf{m})-$ REAL (KIND=nag_wp) array
If nag_mip_ilp_dense (h02bb) exits with ifail $=0,7$ or 9 , the values in the array bl contain the lower bounds imposed on the integer solution for all the constraints. The first $\mathbf{n}$ elements contain the lower bounds on the variables, and the next $\mathbf{m}$ elements contain the lower bounds for the general linear constraints (if any).

2: $\quad \mathbf{b u}(\mathbf{n}+\mathbf{m})-$ REAL (KIND=nag_wp) array
If nag_mip_ilp_dense (h02bb) exits with ifail $=0,7$ or 9 , the values in the array bu contain the upper bounds imposed on the integer solution for all the constraints. The first $\mathbf{n}$ elements contain the upper bounds on the variables, and the next $\mathbf{m}$ elements contain the upper bounds for the general linear constraints (if any).

3: $\quad$ clamda $(\mathbf{n}+\mathbf{m})$ - REAL (KIND=nag_wp) array
If nag_mip_ilp_dense (h02bb) exits with ifail $=0,7$ or 9 , the values in the array clamda contain the values of the Lagrange-multipliers for each constraint with respect to the current working set. The first $\mathbf{n}$ elements contain the multipliers (reduced costs) for the bound constraints on the variables, and the next $\mathbf{m}$ elements contain the multipliers (shadow costs) for the general linear constraints (if any).

4: $\quad$ istate $(\mathbf{n}+\mathbf{m})$ - INTEGER array
If nag_mip_ilp_dense (h02bb) exits with ifail $=0,7$ or 9 , the values in the array istate indicate the status of the constraints in the working set at an integer solution. Otherwise, istate indicates the composition of the working set at the final iterate. The significance of each possible value of istate $(j)$ is as follows.
istate $(j)$ Meaning
-2 The constraint violates its lower bound by more than tolfes (the feasibility tolerance, see h 02 bb ).
-1 The constraint violates its upper bound by more than tolfes.
0 The constraint is satisfied to within tolfes, but is not in the working set.
1 This inequality constraint is included in the working set at its lower bound.
2 This inequality constraint is included in the working set at its upper bound.
3 This constraint is included in the working set as an equality. This value of istate can occur only when $\mathbf{b l}(j)=\mathbf{b u}(j)$.
4 This corresponds to an integer solution being declared with $x_{j}$ being temporarily fixed at its current value. This value of istate can occur only when ifail $=0,7$ or 9 on exit from h02bb.

5: ifail - INTEGER
ifail $=0$ unless the function detects an error (see Section 5).

## 6 Error Indicators and Warnings

Errors or warnings detected by the function:
ifail $=1$
On entry, $\mathbf{n} \leq 0$,
or $\quad \mathbf{m}<0$.
ifail $=-99$
An unexpected error has been triggered by this routine. Please contact NAG.

$$
\text { ifail }=-399
$$

Your licence key may have expired or may not have been installed correctly.

## ifail $=-999$

Dynamic memory allocation failed.

## 7 Accuracy

Not applicable.

## 8 Further Comments

None.

## 9 Example

One of the applications of integer programming is to the so-called diet problem. Given the nutritional content of a selection of foods, the cost of each food, the amount available of each food and the consumer's minimum daily nutritional requirements, the problem is to find the cheapest combination. This gives rise to the following problem:
minimize

$$
c^{\mathrm{T}} x
$$

subject to

$$
A x \geq b, 0 \leq x \leq u
$$

where

$$
c=\left(\begin{array}{llllll}
3 & 24 & 13 & 9 & 20 & 19
\end{array}\right)^{\mathrm{T}}, x=\left(x_{1}, x_{2}, x_{3}, x_{4}, x_{5}, x_{6}\right)^{\mathrm{T}}
$$

is integer,

$$
A=\left(\begin{array}{rrrrrr}
110 & 205 & 160 & 160 & 420 & 260 \\
4 & 32 & 13 & 8 & 4 & 14 \\
2 & 12 & 54 & 285 & 22 & 80
\end{array}\right), \quad b=\left(\begin{array}{r}
2000 \\
55 \\
800
\end{array}\right)
$$

and

$$
u=\left(\begin{array}{llllll}
4 & 3 & 2 & 8 & 2 & 2
\end{array}\right)^{\mathrm{T}}
$$

The rows of $A$ correspond to energy, protein and calcium and the columns of $A$ correspond to oatmeal, chicken, eggs, milk, pie and bacon respectively.

The following program solves the above problem to obtain the optimal integer solution and then examines the effect of increasing the energy required to 2200 units.

### 9.1 Program Text

function h02bz_example

```
fprintf('h02bz example results\n\n');
% Find vector x of length n that Minimizes c.x subject to:
% bl(l:n) <= x <= bu(l:n) and m linear contraints
% bl(n+1:n+m) <= Ax <= bu(n+1:n_m)
n = nag_int(6);
m = nag_int(3);
a = [110, 205, 160, 160, 420, 260;
4,
```



```
bu = [ 4; 3; 2; 8; 2; 2; 1e+20; 1e+20; 1e+20];
% All of x is integer.
intvar = nag_int([1; 1; 1; 1; 1; 1]);
c = [3; 24; 13; 9; 20; 19];
% Initial guess
x = zeros(6,1);
itmax = nag_int(0);
msglvl = nag_int(0);
maxnod = nag_int(0);
intfst = nag_int(0);
toliv = 0;
tolfes = 0;
bigbnd = 1e+20;
[itmax, toliv, tolfes, bigbnd, x, objmip, iwork, rwork, ifail] = ...
h02bb( ...
    itmax, msglvl, a, bl, bu, intvar, c, maxnod, ...
    intfst, toliv, tolfes, bigbnd, x);
% Extract further solution information and display
[bl, bu, clamda, istate, ifail] = ...
h02bz(n, m, iwork, rwork);
fprintf('Final IP objective value = %11.4f\n\n',objmip)
fprintf('%6s%8s%8s%16s%14s%13s\n','Varbl','State','Value',...
    'Lower Bound','Upper Bound','Lagr Mult');
chstate = ['VL';'VU';'FR';'LL';'UU';'EQ';'TF'];
vch = ['Oatmeal';'Chicken';'Eggs ';'Milk ';'Pie ';'Bacon '];
cch = ['Energy ';'Protein';'Calcium'];
for i=1:n
    ich = double(istate(i)) + 3;
    fprintf('%7s%6s%10.2f%14.2f%14.2f%13.2f\n',vch(i,:),...
                                    chstate(ich,:),x(i),bl(i),bu(i),clamda(i))
end
fprintf('\n%6s%8s%8s%16s%14s%13s\n','L Con','State','Value',...
            'Lower Bound','Upper Bound','Lagr Mult');
y = a*x;
for i=n+1:n+m
    ich = double(istate(i)) + 3;
    fprintf('%7s%6s%10.2f%14.2f%14.2e%13.2f\n',cch(i-n,:),...
                            chstate(ich,:),y(i-n),bl(i),bu(i),clamda(i))
end
```


### 9.2 Program Results

| Final IP | object | ve value | 97.0000 |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Varbl | State | Value | Lower Bound | Upper Bound | Lagr Mult |
| Oatmeal | EQ | 4.00 | 4.00 | 4.00 | 3.00 |
| Chicken | LL | 0.00 | 0.00 | 3.00 | 24.00 |
| Eggs | LL | 0.00 | 0.00 | 2.00 | 13.00 |
| Milk | LL | 5.00 | 5.00 | 8.00 | 9.00 |
| Pie | EQ | 2.00 | 2.00 | 2.00 | 20.00 |
| Bacon | LL | 0.00 | 0.00 | 2.00 | 19.00 |
| L Con | State | Value | Lower Bound | Upper Bound | Lagr Mult |
| Energy | FR | 2080.00 | 2000.00 | $1.00 \mathrm{e}+20$ | 0.00 |
| Protein | FR | 64.00 | 55.00 | $1.00 \mathrm{e}+20$ | 0.00 |
| Calcium | FR | 1477.00 | 800.00 | $1.00 \mathrm{e}+20$ | 0.00 |

