HSBC EQUITY QUANTITATIVE RESEARCH

Portfolio Maximum Entropy and Sampling Error Control

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NAG Wilmott Quant Event

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Diversification – Naïve, Classical, Quantum

- **Naïve**

  "1/3 in Land, 1/3 in merchandise, 1/3 in ready to hand" – Rabbi Isaak bar Aha (IV century AD, Babylonian Talmud - quoted in V De Miguel, L Garlappi, R Uppal, see reference below)– **A craftsman solution**

- **Classical –** why investing equally in ***good*** and ***bad*** ? ***Experience + Theory = Science***

  Science + Efficiency = Industry

U. S. AIR FORCE

PROJECT RAND

RESEARCH MEMORANDUM

THE OPTIMIZATION OF QUADRATIC FUNCTIONS

SUBJECT TO LINEAR CONSTRAINTS

Harry Markowitz

RM-1438

21 February 1955

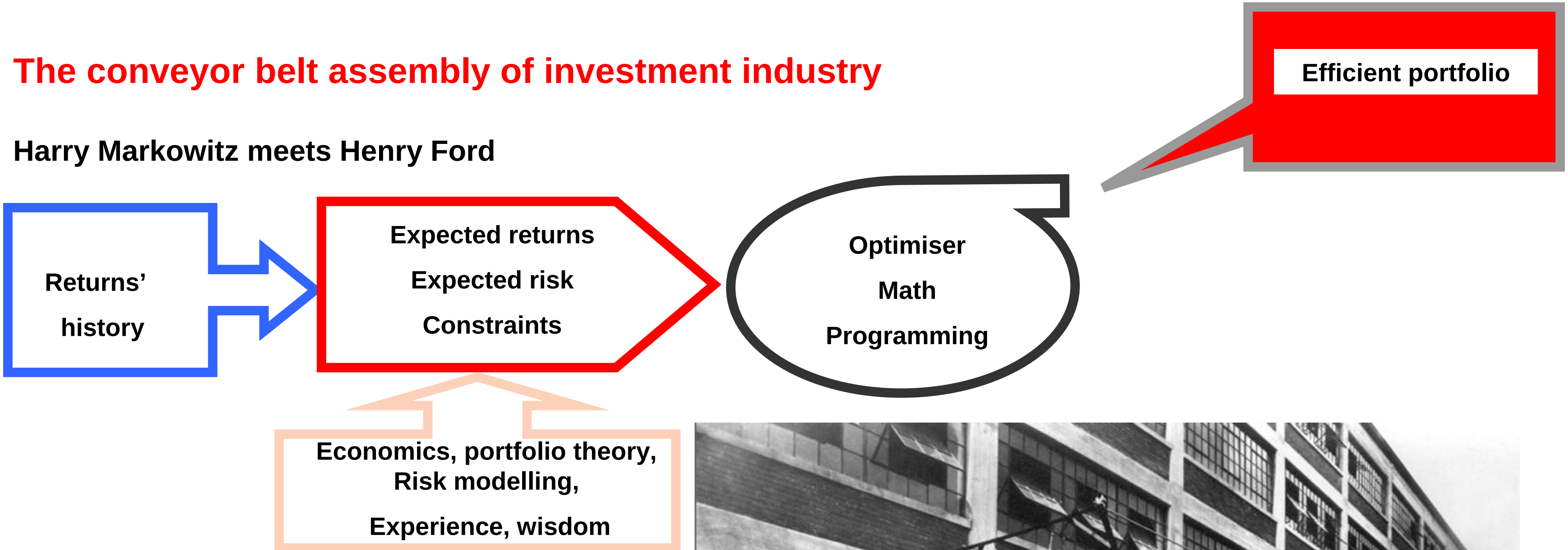
- **Quantum –** the uncertainty is real (Principal reference – A K Bera, and S Y Park, S Y Optimal Portfolio Diversification Using the Maximum Entropy Principle. *Econometric Reviews*, v. 27, p 484, 2008)

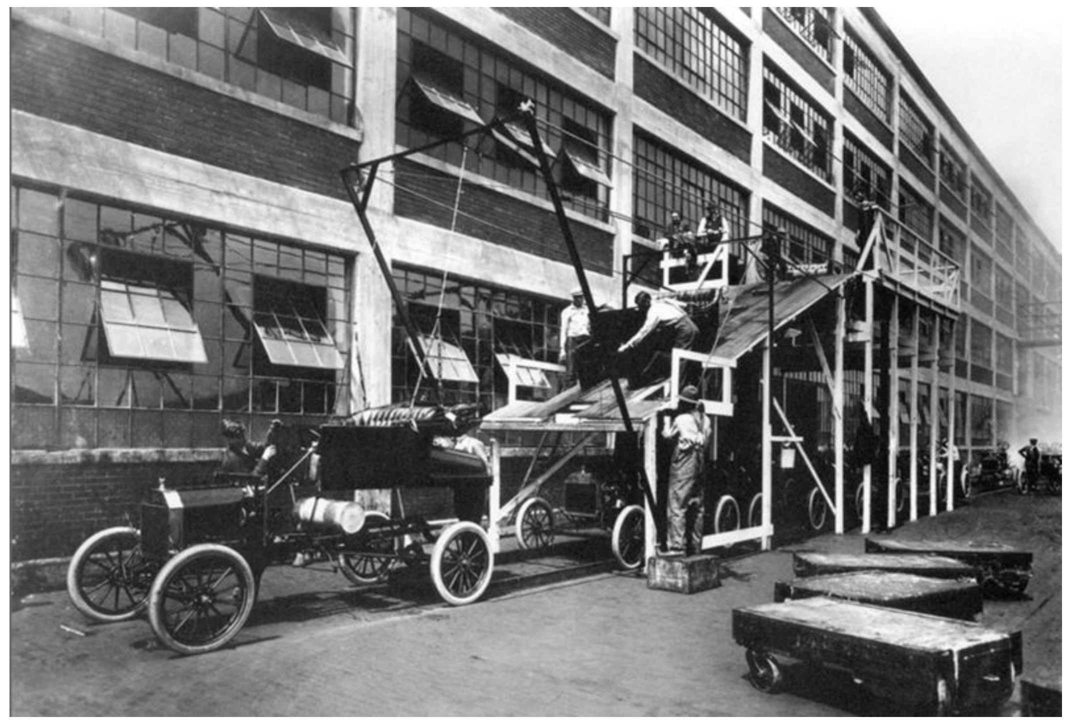
Optimisation – taking investment from craft to industry

- **Objective** – maximum future value of invested capital
  - The concept of efficiency – not all portfolios are equal, some are worse – those with the same expected return but with higher risk
  - Optimisation enables investors to avoid these unfortunate choices by selecting the among portfolios with the same expected return the one with the lowest risk
- **Optimisation inputs** – security **expected** returns, **estimated** risk return inter-dependence (covariance) – mainly **estimated** from **history**
- **Optimised portfolio** (in theory) – higher long-term returns, lower transaction costs, accommodates holding restrictions (keeps regulators and trustees happy)

The conveyor belt assembly of investment industry

Harry Markowitz meets Henry Ford





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Post-modern portfolio theory

Not a conspiracy of latent Luddites

- **Richard Michaud** (Financial Analyst Journal, 45, 1989) warns of MV optimisation's tendency to maximise the effects of the errors in the inputs

- **David Turkington** (Canadian Investment review 17 May 2010) although in favour of the optimised portfolio selection, acknowledges studies indicating that equal-weighted allocation of capital is intuitive and rewarding.

- **Victor DeMiguel, Lorenzo Garlappi, Raman Uppal** (Review of Financial Studies, 22 (5), 2009)

Tested 14 models in 7 data sets and found no statistically significant Sharpe ratio outperformance relative to the equally weighted portfolio

Sampling error – the core cause of the problem

Sampled returns represent inadequately the whole ensemble of possible returns.

A sample selected at a different time, under different conditions

might produce significantly different results

Unreliable estimation of input parameters for optimisation

Optimisers following wrong estimations recommend wrong holdings

Minimise dependency on sampling errors

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Limiting sampling errors – constraints

- **Constraints – limiting exposure – not following blindly the advice of the expected return versus expected risk selection**

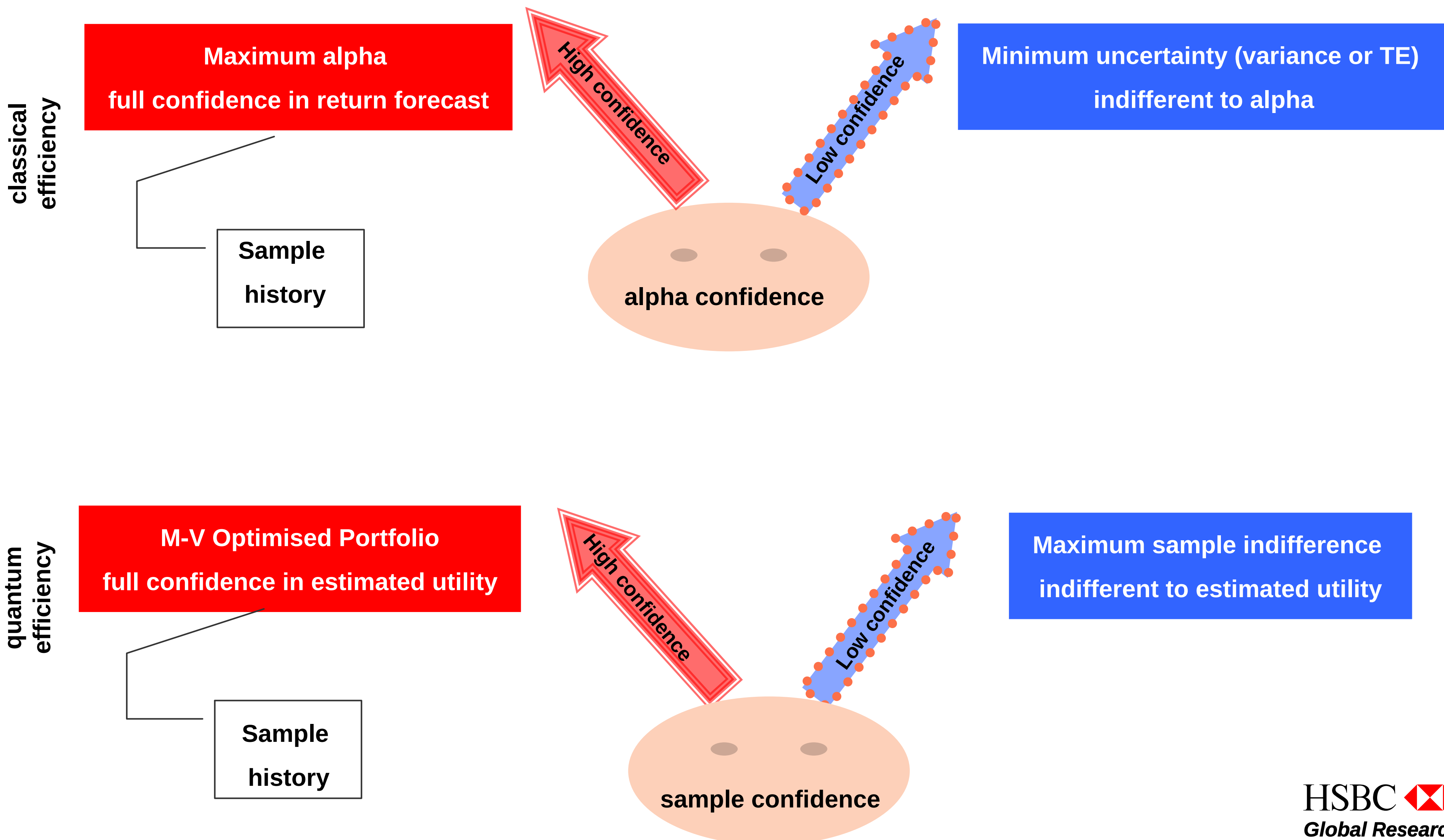
  Return uncertainty requires the risk to be controlled. If not for the uncertainty then the highest expected return ought to be allocated the whole capital available

  Holding constraints reflect uncertainty with respect to the estimation of risk (another way to say ***uncertainty***)

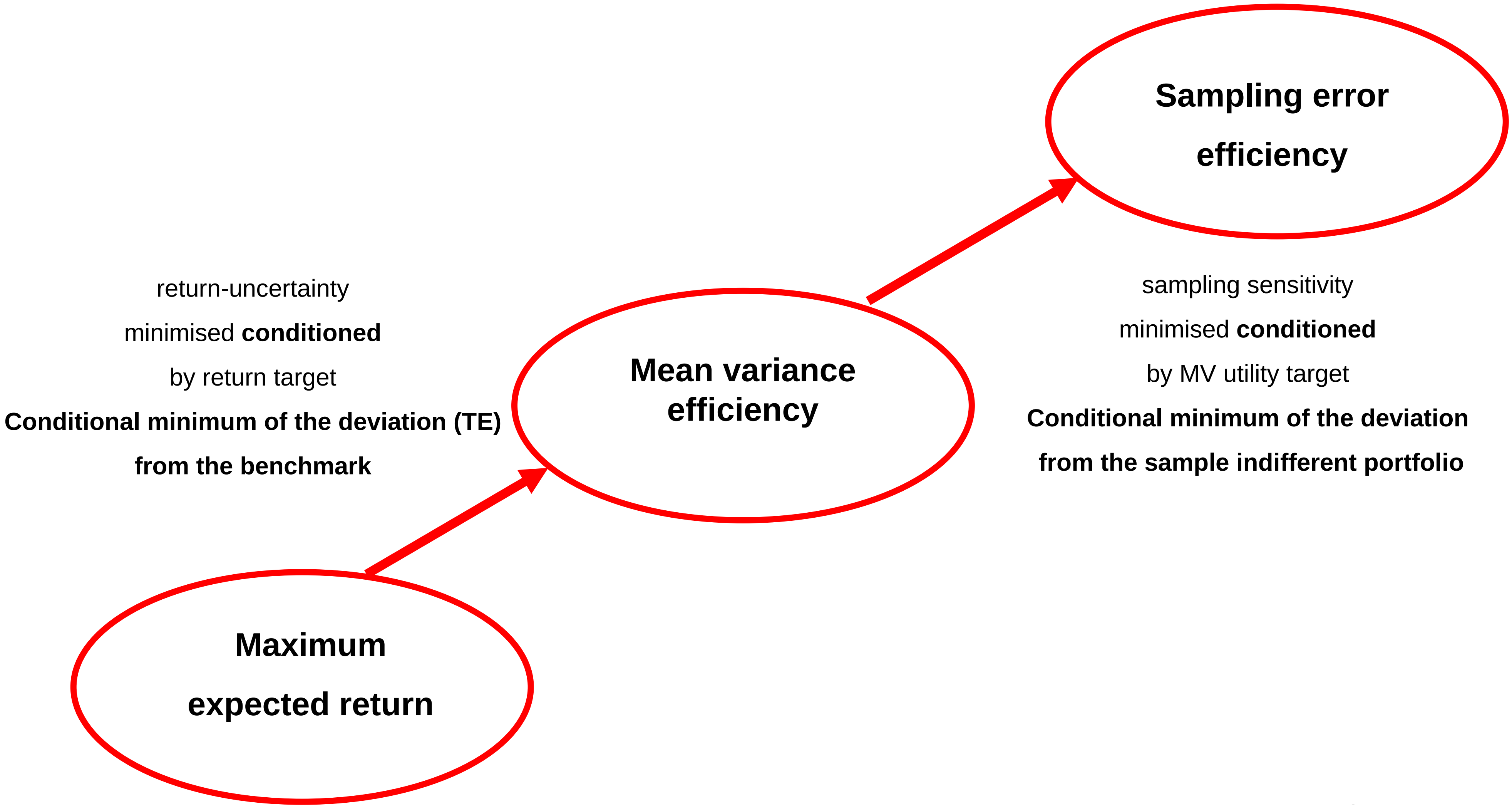
  Equally-weighted diversification expresses ultimate disbelief in the estimate, including estimate of the uncertainty

- Jagannathan and Ma (The Journal of Finance, 58, 4, 2003) – even imposition of wrong in population non-negativity constraints reduces the realised risk (for sample covariance matrix)

Taking uncertainty seriously – quantum correction



Conditional optimisation defines efficiency



Building blocks

- Measure of distance
- Definition of level of confidence
- Definition of the sampling-indifferent portfolio (shrinkage target)

A measure of distance

- Bera and Park (2008) propose to use **Kullback-Leibler cross-entropy**

$$X(w,q) = \sum_{i}^{N} w_i \log\left(\frac{w_i}{q_i}\right)$$

optimised portfolio ***w*** , shrinkage target ***q***

- The sum of weighted logarithms is never negative. It reaches zero for portfolio weights equal to the shrinkage target weights
- For an equally weighted shrinkage target ***X(w,q)*** is the (-) Shannon information entropy
- Additive, global, sample-independent, risk-model independent
- Possible to generalise to short holdings

Defining level of confidence

1. Estimate expected return and risk according to the sample
2. Generate re-sampling of the returns and for each of the re-sampled histories of returns estimate expected return and risk
3. Calculate the optimal mean variance portfolio for the empirical sample and for each of the bootstrap generated virtual histories
4. Substitute **empirical sample mean and variance-covariance into each set of portfolio weights and obtain re-sampling of mean-variance utility. By construction the largest utility is obtained for the optimisation weights of the empirical sample**
5. Estimate the distribution of the maximum utility values of item 4
6. Assess the utility value corresponding to the percentage of confidence – an inverse function of the cumulative distribution

Utility function distribution (from bootstrapping of a minimum variance solution)

Fig 1-a. Utility distribution density

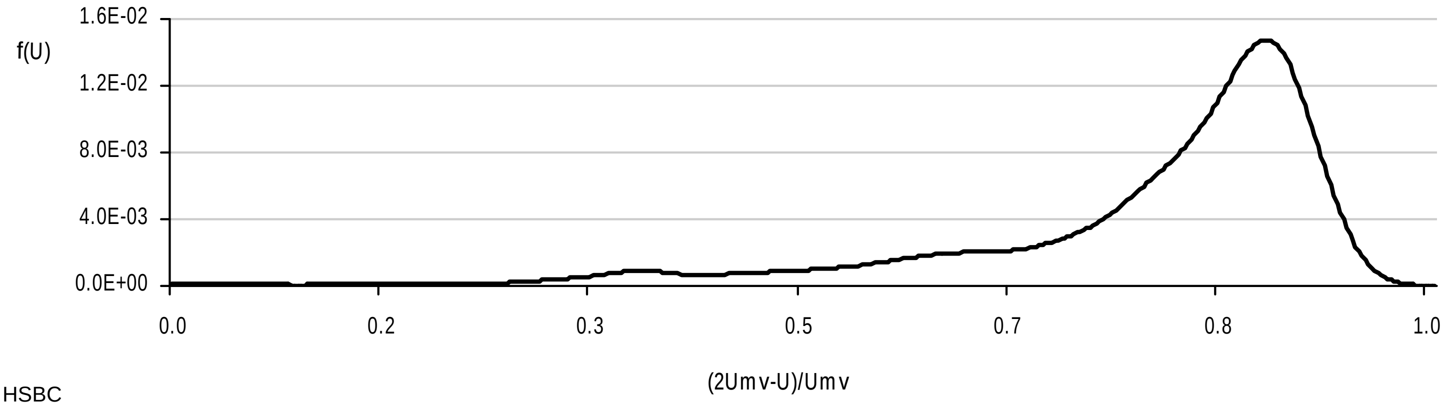


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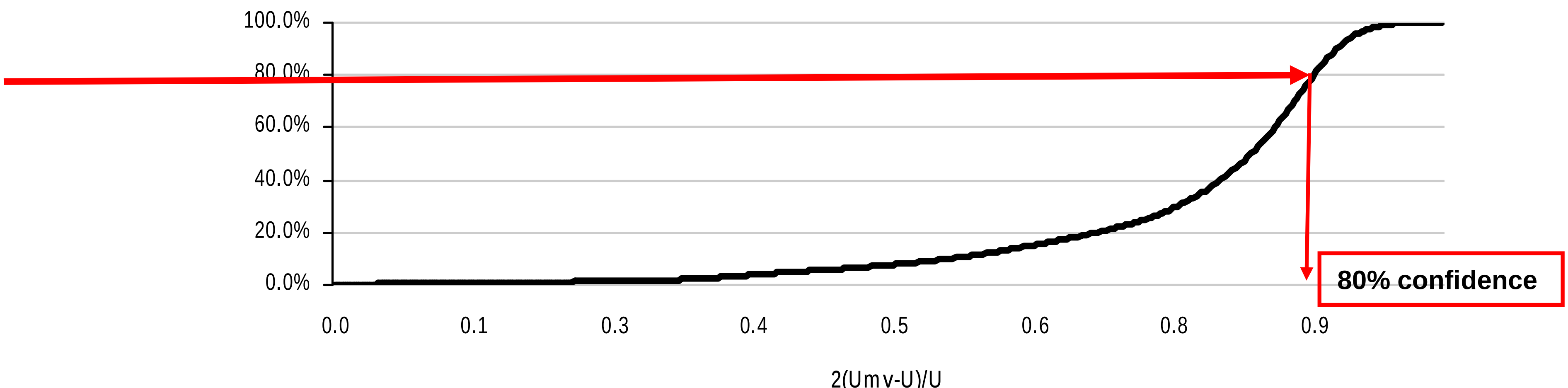
Fig 1-b. Cumulative distribution for confidence interval



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Conditional minimisation of cross-entropy

- For a given shrinkage target q_i minimum cross entropy solution is given by

$$w_{iCE} = \arg\min_{w} \left[\sum w_i \log\left(\frac{w_i}{q_i} \right) \right]$$

subject to the usual holding constraints and to an additional defined by the lowest acceptable value of utility (according to an investor's confidence) $-U_0$

$$w_i \mu_i - \frac{\lambda}{2} w_i \Sigma_{ij} w_j \geq U_0; \quad (\mu, \Sigma)$$ are the sample-estimated returns and covariance matrix

The minimisation is done using NAG routine E04UC

Special thanks for putting together the NAG toolbox for Matlab

Diversification effect of conditional cross-entropy minimisation – country allocation – MSCI country equity indices, sample (60 m) - estimated covariance matrix

Fig 2-a. MV solution

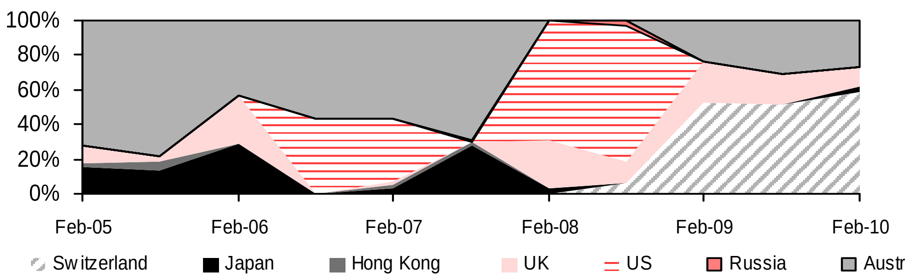


Fig 2-b. 90% confidence

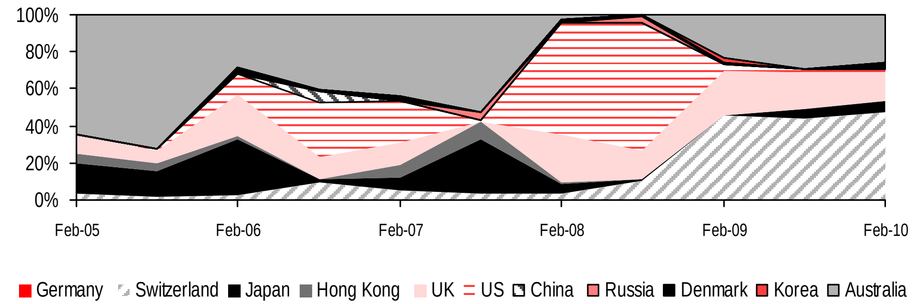


Fig 2-c. 75% confidence

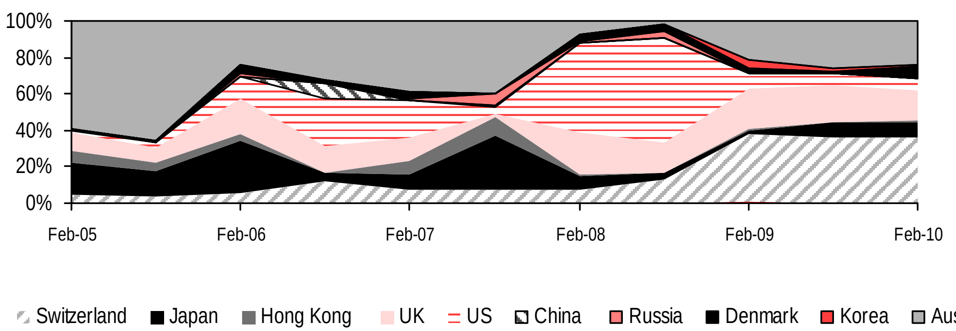
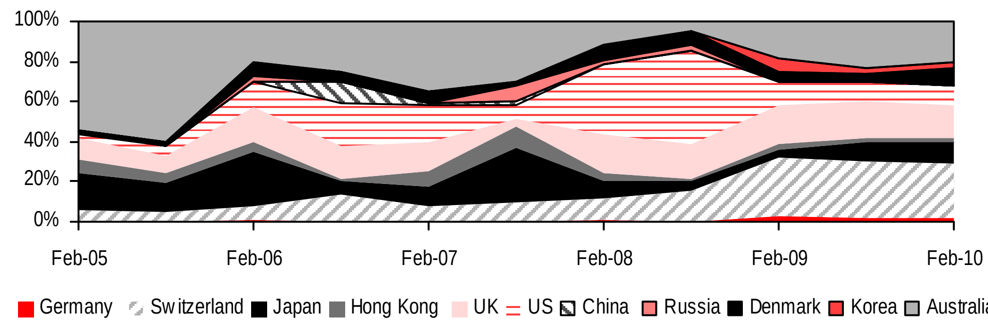


Fig 2-d. 50% confidence



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Small utility costs for increased diversification

Fig 3-a. Estimated utility

(minimum variance optimisation)

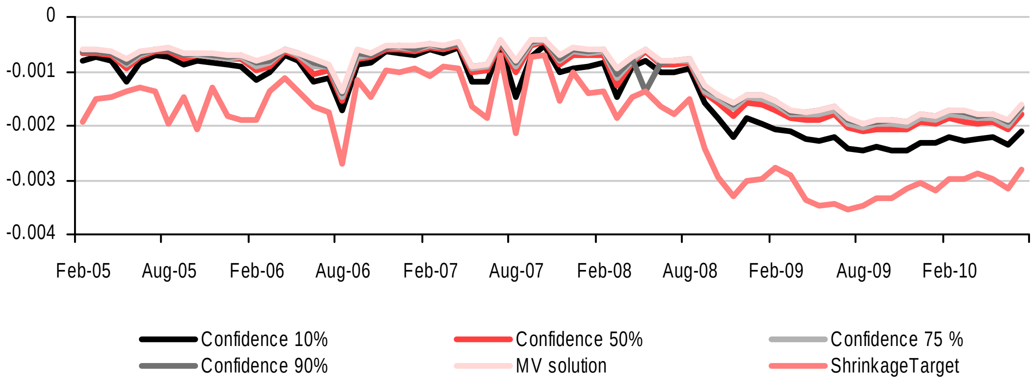
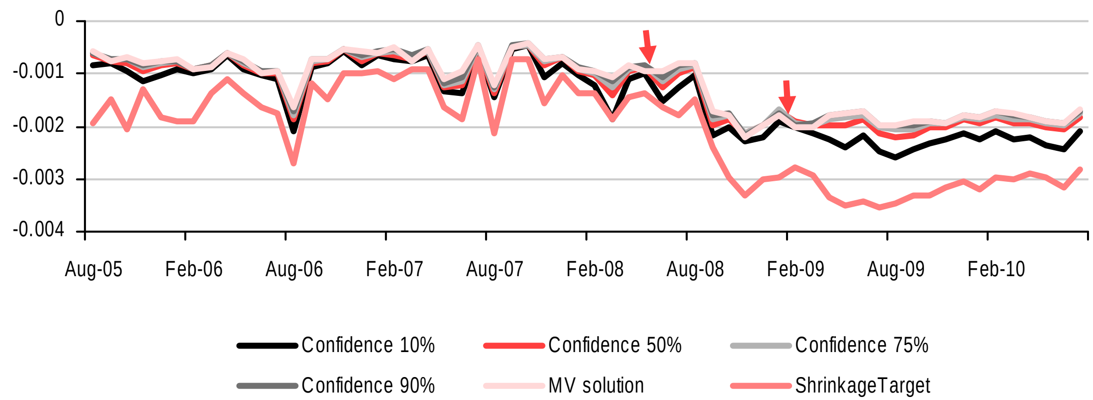


Fig 3-b. 6 months after optimised rebalancing – lower confidence solutions closing the utility gap



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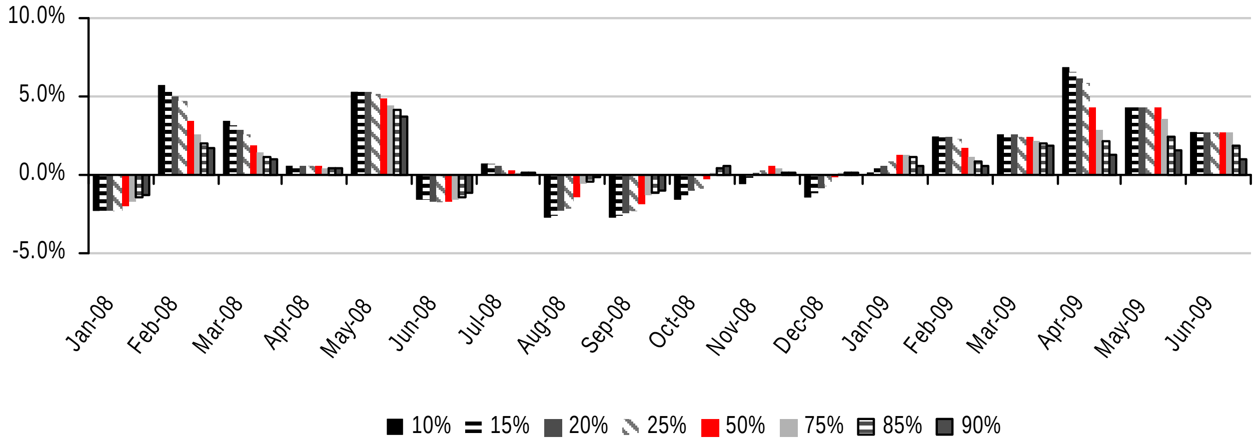
Reduced downside risk – Mean-Variance optimisation and conditional minimum cross-entropy. Comparison of 6-month losses – (positive value – smaller loss)

Fig 4. Conditional minimum cross-entropy (for stated confidence levels) *vs* Mean-Variance



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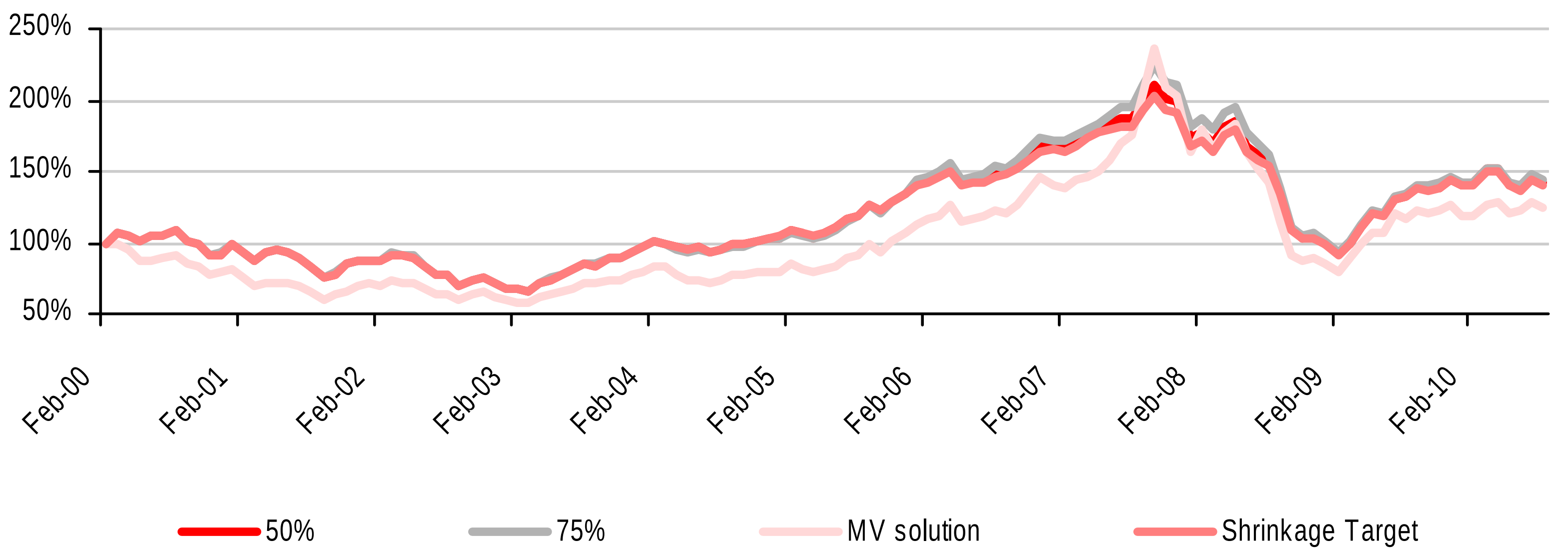
Beyond minimum risk

Fig 5. Ten year performance

Mean-Variance optimisation with expected return defined as exponentially weighted mean of the past 60 months



Source HSBC

Minimum cross-entropy solution captures Mean-Variance gains and moderates its losses

Fig 6-a. Minimum cross-entropy solution performance relative to MV solution

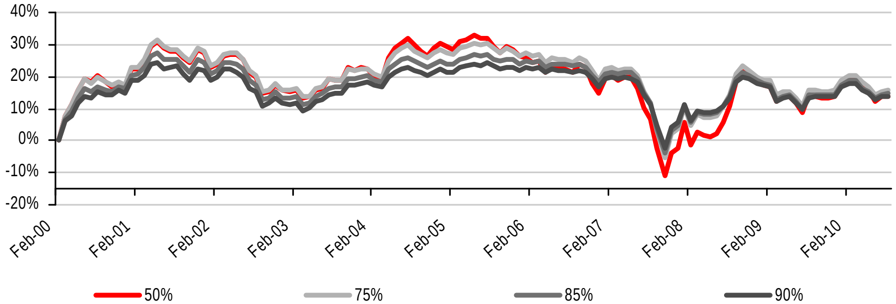
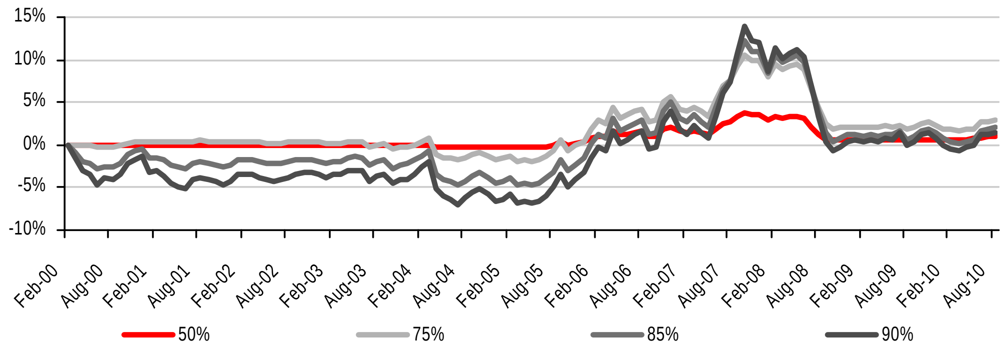


Fig 6-b. Minimum cross-entropy solution performance relative to equally-weighted allocation



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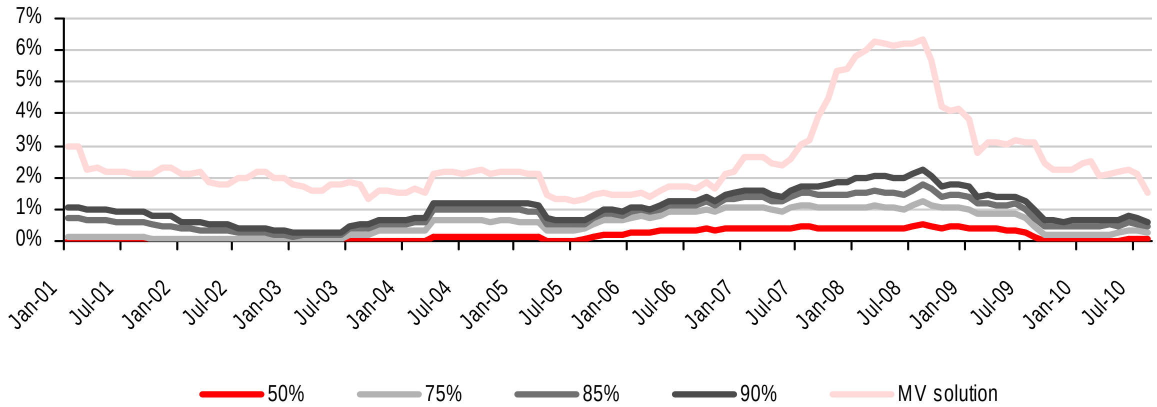
Realised tracking error and information ratio relative to the equally weighted allocation

Fig 7-a. TE



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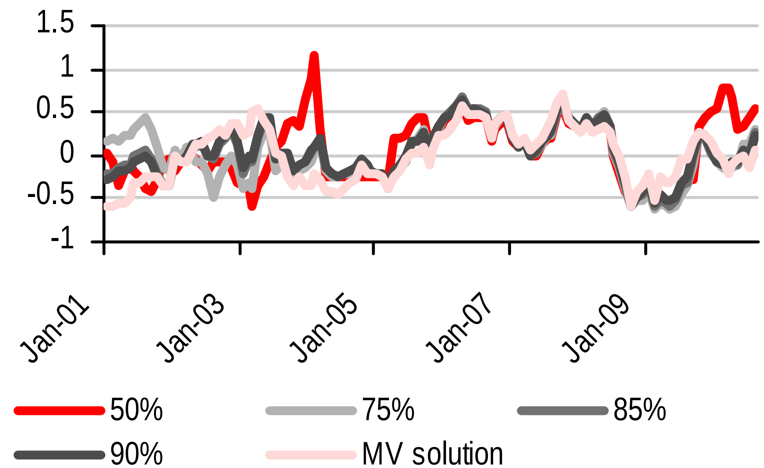
Positive IR of the MV (useful sample information) is maintained in the minimum cross-entropy solution

Fig.7-b

IR



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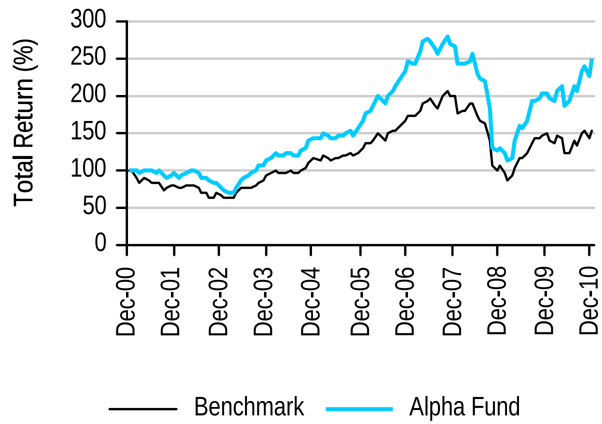
Stock level portfolio optimisation – a European Value-Momentum fund

- Return expectation – combined earnings yield and three months price momentum
- Benchmark weights for standard optimisation – DJ Stoxx 600
- Market cap-weighted benchmark underperforms Alpha (top decile selected) fund (Fig. 8)
- Comparison of Alpha with equally weighted allocation of Stoxx 600 constituents (EW) indicates negative market cap bias (Fig.9)

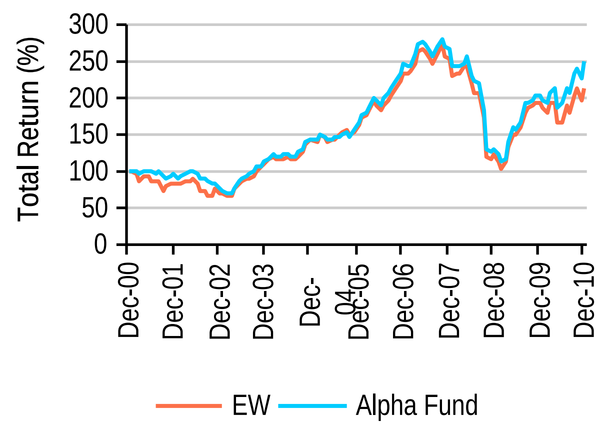
Fig. 8



	Benchmark	Alpha Fund
Average return p/a	4%	10%

Source: HSBC

Fig. 9



	EW	Alpha Fund
Average return p/a	8%	10%

Source: HSBC

Mean-variance optimisation

- ➢Small enhancement of performance (Fig.10)
- ➢High estimated tracking error, which nonetheless understates the realised error (Fig.11)
- ➢Tuning to decrease tracking error will bring the performance even nearer to the weak benchmark

Fig. 10

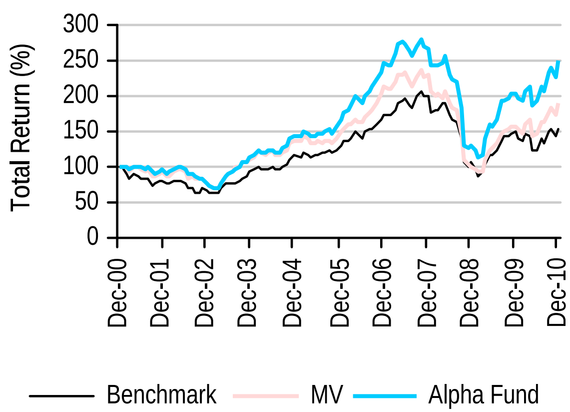
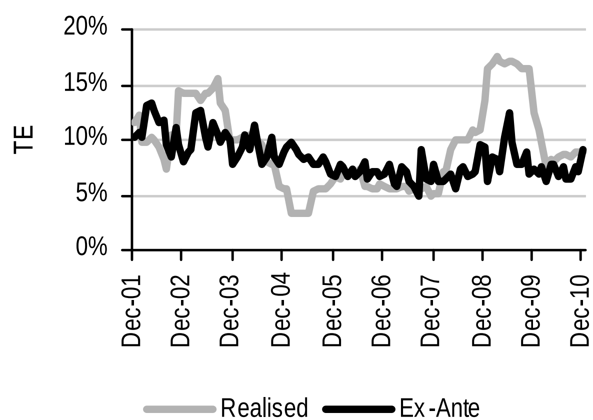


Fig. 11



	Benchmark	MV	Alpha Fund
Average return p/a	4%	7%	10%

Source: HSBC

Source: HSBC

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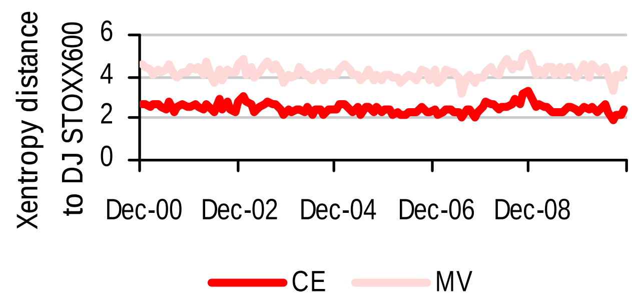
Enhanced diversification with equally-weighted target

- Sample ‘*indifferent*’ target – equally weighted (weakening market cap bias and sample sensitivity)
- Utility limit 85% of the mean-variance optimised portfolio
- CE solution – minimum *(cross-entropy)* distance to the equally weighted target that satisfies at least 85% of mean-variance (maximum possible) utility

Fig. 12 – CE solution closer to the Stoxx DJ 600 benchmark



Source: HSBC

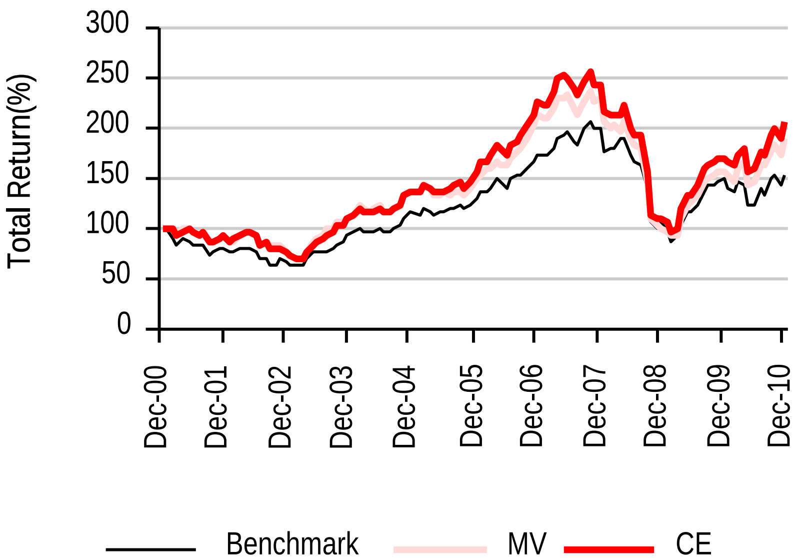
- Portfolio variance is estimated using factor risk model (Northfield).

- Minimum required utility limit is arbitrary. Could not re-generate risk model for re-sampled returns
- From previous example – higher limit corresponds to higher confidence, maximum utility – MV solution 100% confidence
- Outlook for future research – shrunk (*e.g.* Ledoit-Wolf method) sample covariance matrix

Performance – short and long term

Fig 13-a.

1 m re-balancing

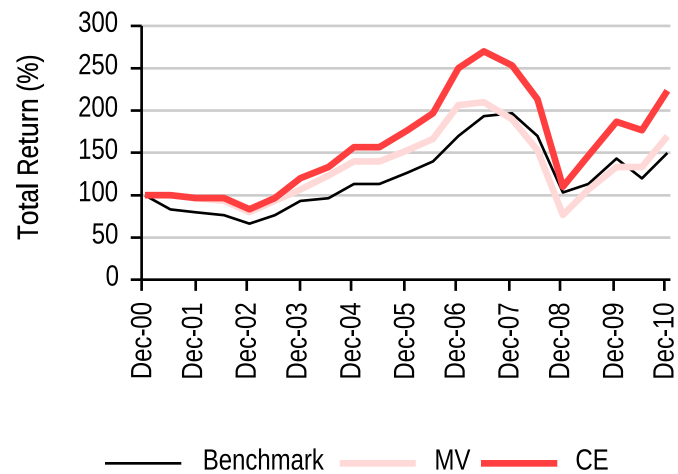


	Benchmark	MV	CE
Average return p/a	4%	7%	8%

Source: HSBC

Fig. 13-b.

6 m re-balancing



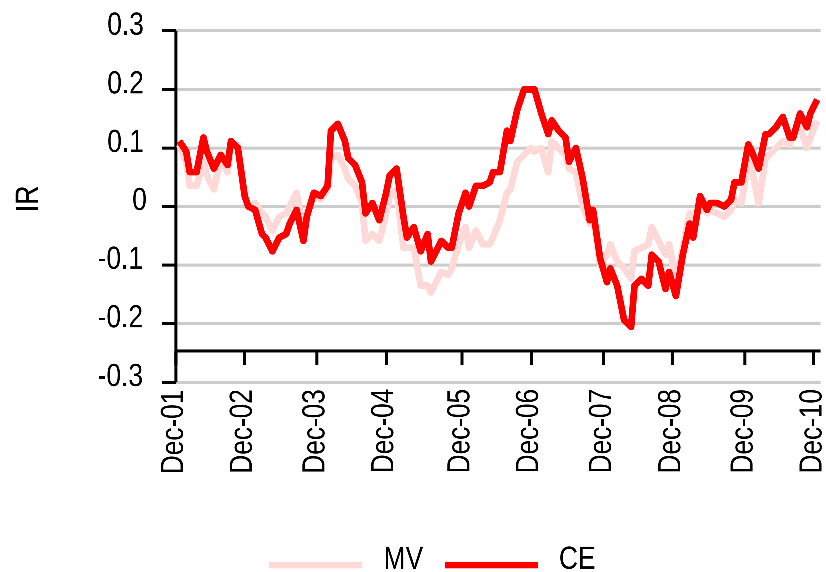
	Benchmark	MV	CE
Average return p/a	4%	5%	8%

Source: HSBC

Alpha information maintained

➢The proposed solution maintains essential stock selection and risk modelling information

Fig 14 – ***Realised*** information ratio, for monthly returns



Source: HSBC

Summary

I. **Ideology** – extension of Markowitz' programme – equilibrium between the desired and the feared, between classical efficiency and indifference to estimation errors;

first quantum correction (hopefully)

II. **Practicality**

1. Introduction of a sample independent measure of difference between two distributions of capital
2. Shrinkage applied directly to portfolio weights – can be used in testing and comparing risk models
3. Strict multiple holding constraints can be replaced by a single **GLOBAL** limit on cross-entropy

   (a straightforward extension of optimiser objective function)

III. **Results**

1. Enhanced diversification by introducing a single global constraint on a minimum acceptable value of utility
2. Preservation of the essential alpha information
3. Reduction of realised risk and enhanced out-of-sample performance

Disclosure appendix

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