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Manchester Business School

Interest Rate Models: An ALM Perspective

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Interest Rate Models: An ALM Perspective (with NAG implementation)

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Full paper: http://www.personal.mbs.ac.uk/ser-huang-poon/

Joint work with Eric Guan, Bing Gan and Aisha Khan
All postgraduate students at MBS
MSc QFFE (Quantitative Finance and Financial Engineering) Industry Linked Project

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QFFE Course Structure

<u>Semester I: 3 months</u>

- Asset Pricing Theory
- Derivative Securities
- Stochastic Calculus
- VBA, C++

<u>Optional units</u>

- Portfolio Investment (Barra)
- Macro and International Investment
- Martingale Process

<u>Semester II: 3 months</u>

- Financial Econometrics
- Interest Rate & Credit Derivatives

<u>Optional units</u>

- Credit Risk Modelling & Management
- Computational Finance
- Mathematical Modelling of Finance
- Real Options

Summer Dissertation: June-July, Preparation starts from Spring

Submission of thesis in first week of September

Project Remit

- Proposed by Group ALM of ABN AMRO in Amsterdam
- How sensitive is model choice to ALM outcome
  - Consider switching from BK to HW
  - Question if they should move to more sophisticated model: LMM or SMM
- No access to bank's confidential data
- Four contesting models:
  - Hull-White (HW)
  - Black-Karasinski (BK)
  - Libor Market Model (LMM) and
  - Swap Market Model (SMM)
- Test 20 annual hedging performance of 10x1 Bermudan swaption of the single factor version of the four contesting models on EURO and USD.

Different Types of Interest Rates

- Short rate

$$dr = (\theta(t) - ar)dt + \sigma dz$$

- Forward rate
- Libor rate

$$dL_i(t) = \mu_i(t)L_i(t)dt + \sigma_i(t)^T L_i(t)dW(t)$$
$$0 \le t \le T_i, \quad i = 1, 2, \ldots N$$

$$dS_{n,m}(t) = S_{n,m}(t)\sigma_{n,m}(t)dW_t^{n,m}$$

- Swap rate
  - Fixed strike in exchange for a series of floating rates
  - Payer vs. Receiver

[No arbitrage condition must hold for all interest rates and all interest rate products]

Cash Flows and Rates

$$
\begin{array}{ccccccccc}
0 & \text{---} & 1 & \text{---} & 2 & \text{---}\cdots\text{---} & t & \text{---}\cdots\text{---} & T \\
r_0 & & r_1 & & r_2 & & r_t & & r_T
\end{array}
$$

$$
\begin{array}{ccccccc}
f_0(0) & & & & & & \\
f_1(0) & f_1(1) & & & & & \\
f_2(0) & f_2(1) & f_2(2) & & & & \\
\vdots & & & & & & \\
f_T(0) & f_T(1) & f_T(2) & \cdots & f_T(t) & \cdots & f_T(T)
\end{array}
$$

$$
\begin{array}{ccccccc}
S_{0,T}(0) & & & & & & \\
S_{1,T}(0) & S_{1,T}(1) & & & & & \\
S_{2,T}(0) & S_{2,T}(1) & S_{2,T}(2) & & & & \\
\vdots & & & & & & \\
S_{T-1,T}(0) & S_{T-1,T}(1) & S_{T-1,T}(2) & \cdots & S_{n,m}(t) & \cdots & S_{T-1,T}(T)
\end{array}
$$

Plain Vanilla Products

- Yield curve, Y(T)
- Bond prices
- Forward rate agreement (FRA)
- Caplet, floorlet, cap, floor
- Swap, forward starting swap
- Swaption: European vs. Bermudan

Price quotation in Black Vol

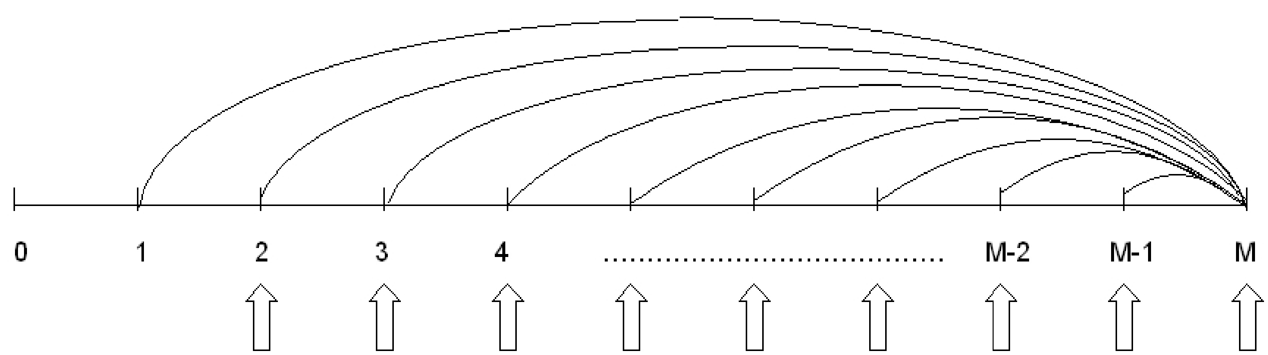
- Only for European (ATM) products and under the terminal measure
- Bloomberg gives implied vol for Gaussian model also

Swap Tenor	Swaption Maturities									
Mean	1	2	3	4	5	6	7	8	9	10
1	19.63%	19.39%	18.86%	18.12%	17.38%		16.03%			14.50%
2	19.44%	18.99%	18.36%	17.62%	16.85%		15.52%			
3	19.09%	18.49%	17.81%	17.07%	16.36%		15.12%			
4	18.74%	17.93%	17.22%	16.54%	15.90%		14.76%			
5	18.26%	17.38%	16.69%	16.05%	15.48%					
6	17.67%	16.88%	16.24%	15.68%	15.19%					
7	17.13%	16.43%	15.90%	15.40%						
8	16.66%	16.06%	15.59%							
9	16.23%	15.72%								
10	15.85%									

$$PSwaption_{t,T} = \delta \sum_{j=n+1}^{N+1} B_{t,T_j} \left[S(t,T_i,T_N) N(d_1) - kN(d_2) \right]$$

Bermudan Swaption

- Constant maturity swap vs. Fixed tail swap
- Fixed tail swap resembles home mortgage with prepayment feature
- Fixed tail swap can be hedged using 10x1 co-terminal European swaptions



Choice of Calibrated Instruments

- Guided by no-arbitrage or hedging trades
- Rates fixed using bond prices or FRA
- Volatility fixed using caplet or European swaptions
- The choice of calibrated instruments determines the complexity of optimisation procedure and the time needed to reach convergence.

Swap Tenor	Swaption Maturities									
Mean	1	2	3	4	5	6	7	8	9	10
1	19.63%	19.39%	18.86%	18.12%	17.38%		16.03%			14.50%
2	19.44%	18.99%	18.36%	17.62%	16.85%		15.52%			
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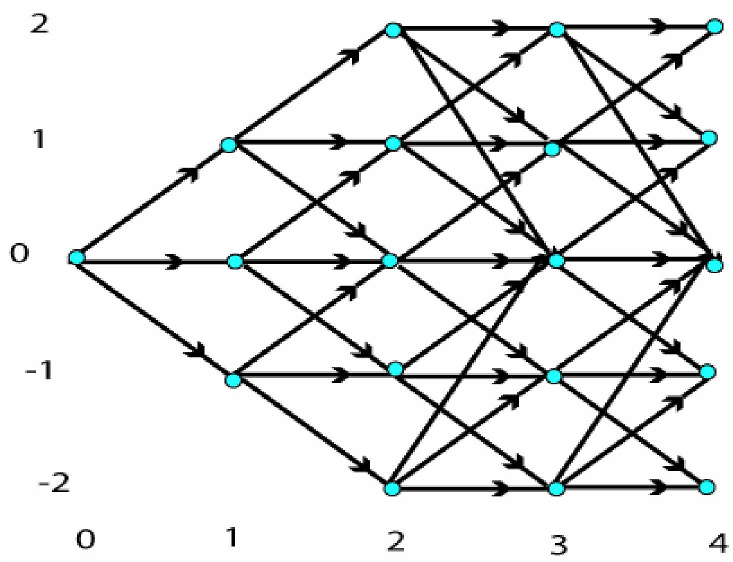
7	17.13%	16.43%	15.90%	15.40%						
8	16.66%	16.06%	15.59%							
9	16.23%	15.72%								
10	15.85%									

Short Rate Trinomial Tree

$$dr = (\theta(t) - ar)dt + \sigma dz$$



- Time step is 0.1 year, 11year=110 time steps
- θ(t) calibrated to bond prices
- Mean reversion rate a(t)->a, estimated from time series data
- σ(t) follows 3-point interpolation

Simulate LMM and SMM under T(n+1) measure

$$\frac{dL_i(t)}{L_i(t)} = -\sum_{j=i+1}^{N} \frac{\delta_j L_j(t)\sigma_{ij}(t)}{1+\delta_j L_j(t)} dt + \sigma_i(t) \cdot dW^{Q^T_{n+1}}(t)$$

$$dS_{n,M}(t) = S_{n,M}(t)\sigma_{n,M}(t)dW_t^M + drift$$

$$\mu_n = \sum_{i=n}^{M-2} \left(\tau_{i+1} \frac{C_{i+1,M}}{C_{M-1,M}} \rho_{n,i+1} S_{n,M} \sigma_{n,M} S_{i+1,M} \sigma_{i+1,M} \prod_{j=n+1}^{i} (1+\tau_j S_{j,M}) \right)$$

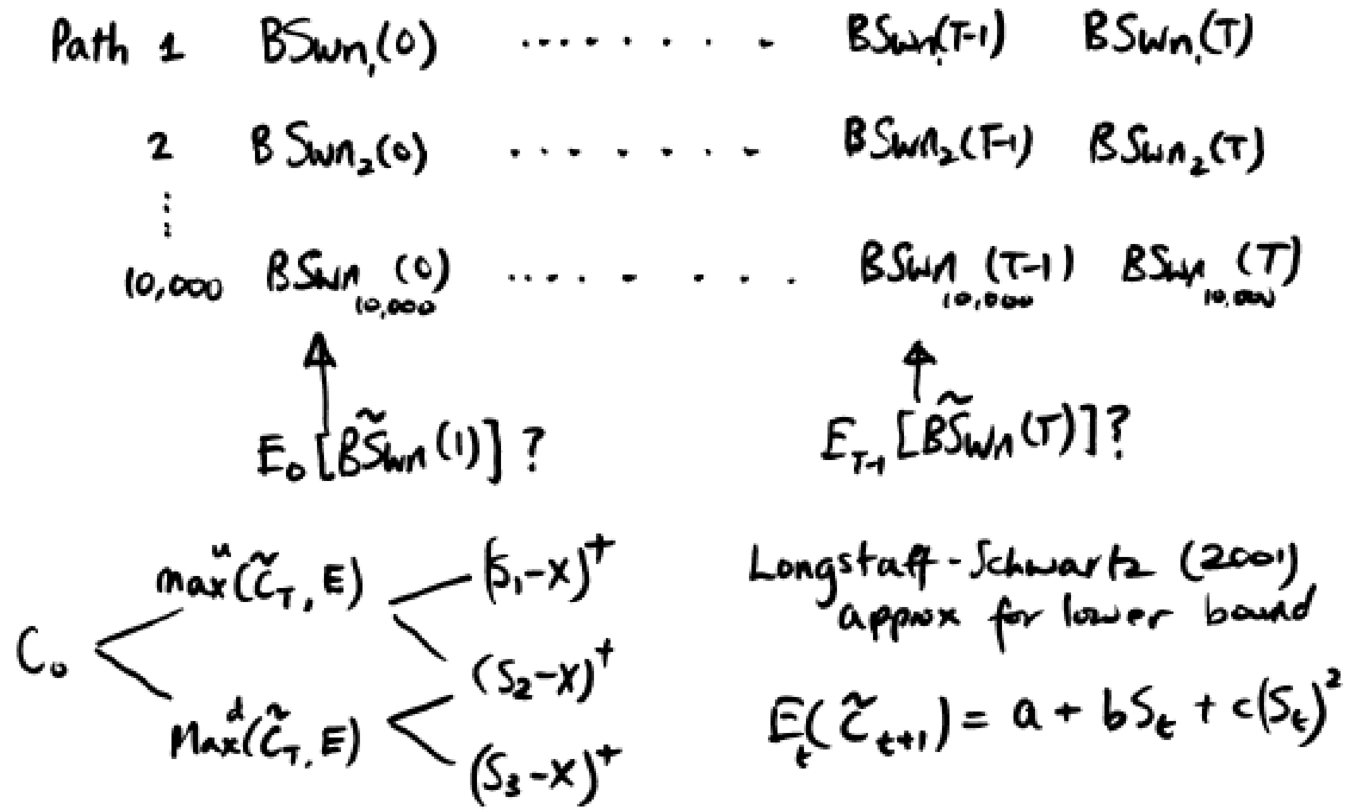
LMM& SMM Simulations

- For each path

$$
\begin{array}{ccccccc}
0 & 1 & 2 & \cdots & t & \cdots & T \\
L_T(0) & L_T(1) & L_T(2) & \cdots & L_T(t) & \cdots & L_T(T) \\
\vdots & & & & & & \\
L_{2,T}(0) & L_{2,T}(1) & L_{2,T}(2) & & & & \\
L_{1,T}(0) & L_{1,T}(1) & & & & & \\
L_{0,T}(0) & & & & & &
\end{array}
$$

- Delta δ=1 yr, 10,000 simulations, single factor model
- Time step δ is not crucial if predictor-corrector method is used to correct the drift
- Use NAG random number generator **g05ddc**

Approximation for early exercise decision



- Call regression routine **g02dac**

Calibration

1. Convert Black Vol into European swaption prices using **g01eac** (prob for std normal distribution).

2. Build short rate HW, estimate σ_0, σ_3 and σ_{11} by minimising RMSPE of 10 co-terminal European swaptions using NAG **e04unc** (solving nonlinear least square using sequential QP).

3. Use the calibrated model to calculate 10x1 Bermudan swaption prices by stepping backward through the tree.

4. Build short rate BK; repeat steps 2-3 for lognormal rate.

Calibration (cont'd)

5. LMM calibrated to European swaption

Use Rebonato's approximation of swaption vol

$$\left(v_{n,M}^{LFM}\right)^2 = \sum_{i,j=\alpha+1}^{\beta} \frac{\omega_i(0)\,\omega_j(0)\,L_i(0)\,L_j(0)\,\rho_{ij}}{T_n\left[S_{n,M}(0)\right]^2} \int_0^{T_n} \sigma_i(t)\,\sigma_j(t)\,dt$$

where $v_{n,M}^{LFM}$ is a proxy for Black volatility $v_{n,M}(T_n)$.

$$\begin{aligned} \sigma_k &= \phi_k \times \varphi_k(a,b,c,d) \\ \varphi_k(a,b,c,d) &= \left[a(T_k - t) + b\right] e^{-c(T_k - t)} + d \end{aligned}$$

Call Nag **e04unc** to minimise vol errors for 10 rates instead of minimising 10 pricing errors using 10,000 simulation for each loop

Calibration (cont'd)

(Cont'd LMM)

- 10,000 simulation (**g05ddc**) to calculate European swaption prices,
- 10 regressions (each with 10,000 observations, **g02dac**) to derive Bermudan prices.

6. SMM; same as step 5 but for SMM. Need additional step to solve simultaneous equations (**f07adc** LU factorization, and **f07aec** solution of real sys of linear equations) in order to derive the discount factor.

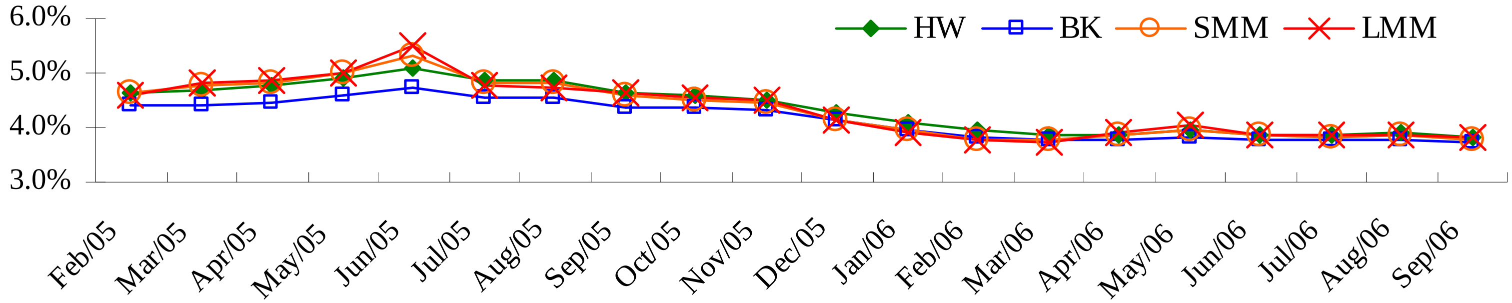
7. Repeat steps 2-6 for pricing 9x1 swaption.

8. Repeat steps 2-7 20 times through the sample period.

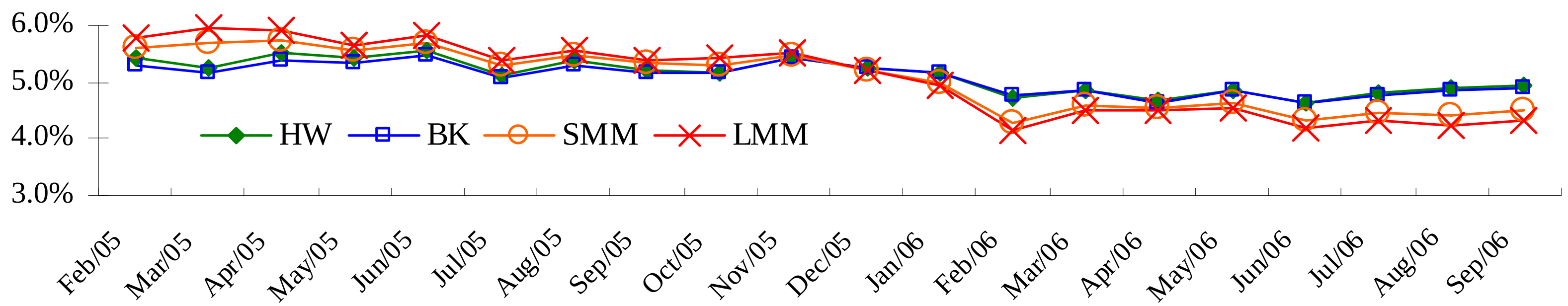
9. Do it once for Euro rate; repeat the whole process for USD interest rate; supposed to calibrate to Brazilian data, but no appropriate market data.

11-Y Bermudan Swaption Prices

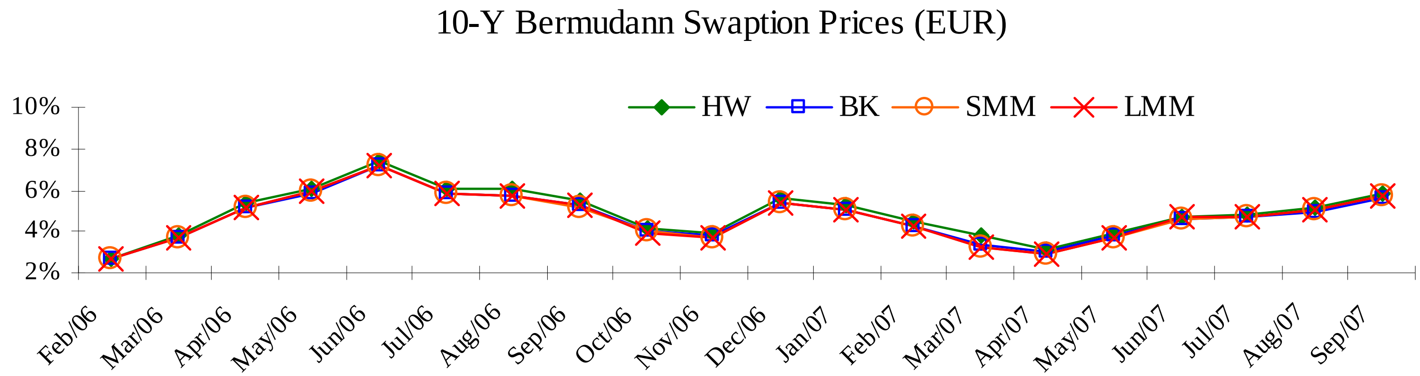
11Y Bermudan Swaption Prices (EUR)

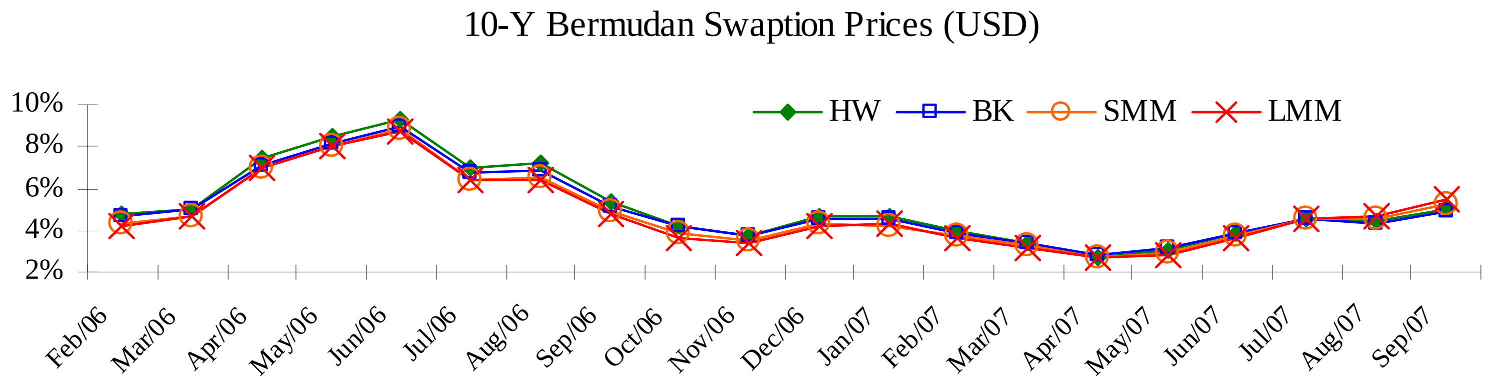


11Y Bermudan Swaption Prices (USD)

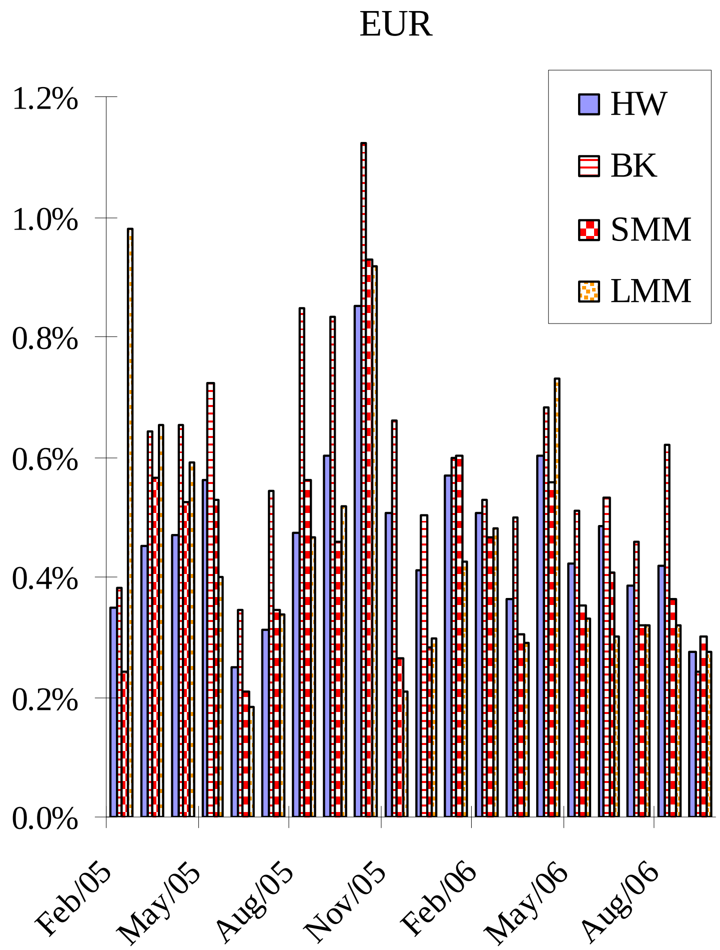


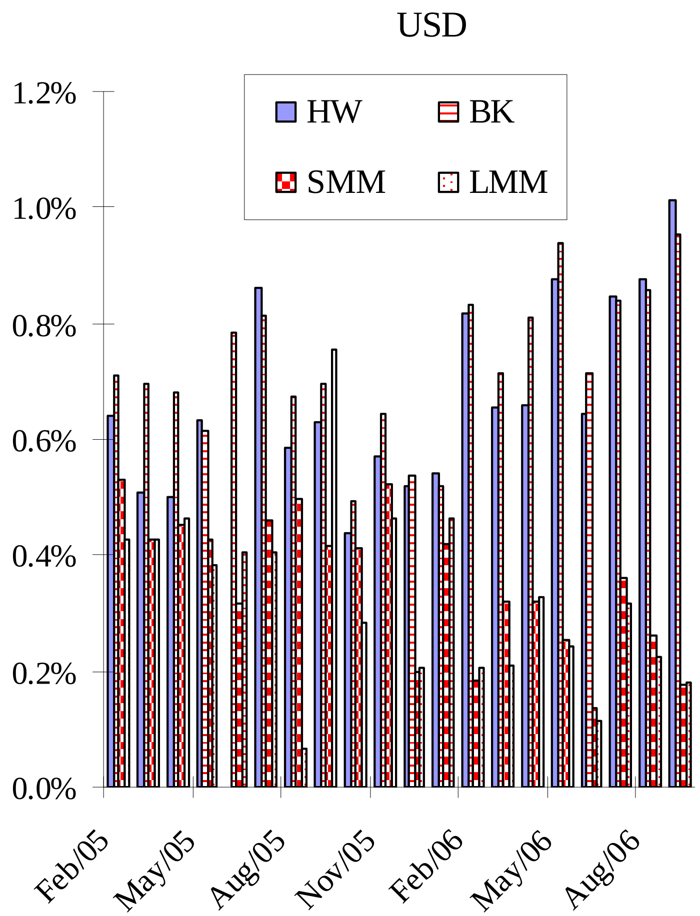
10-Y Bermudan Swaption Prices



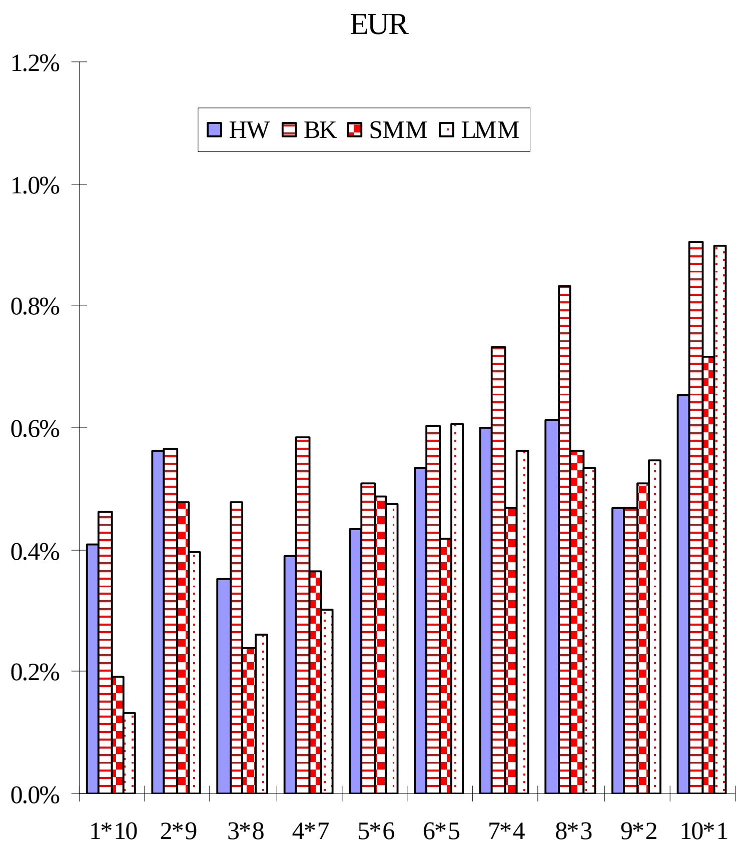


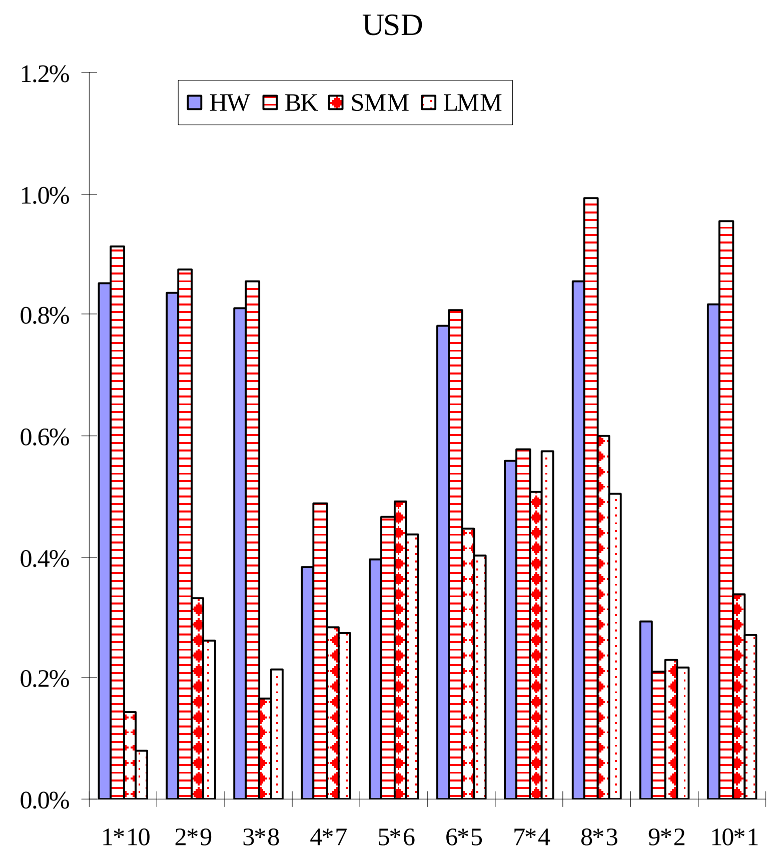
RMSE (Pricing Error) by Date

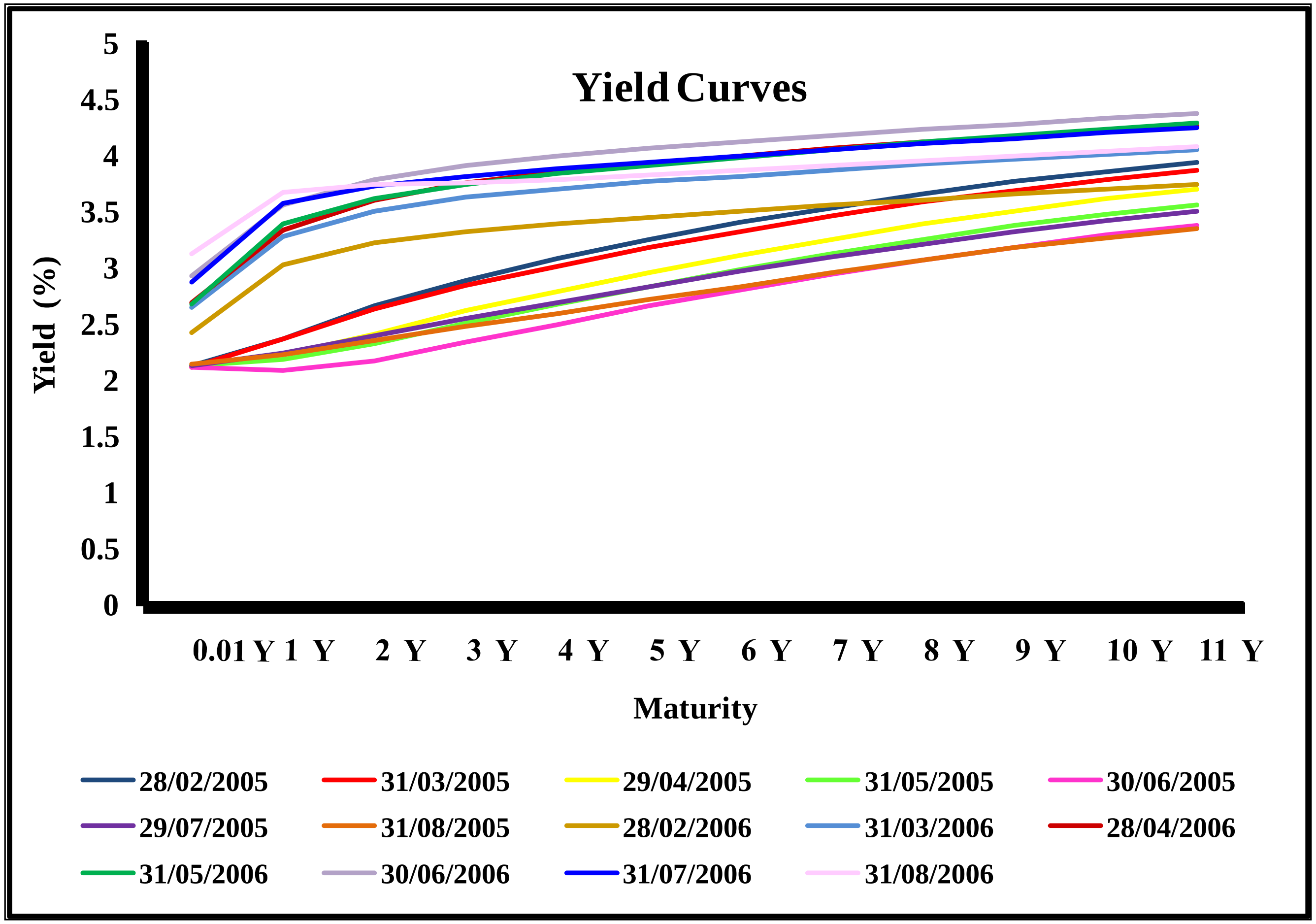




RMSE (Pricing Error) by Contract






Yield Curves
Yield (%)
5
4.5
4
3.5
3
2.5
2
1.5
1
0.5
0
0.01 Y
1 Y
2 Y
3 Y
4 Y
5 Y
6 Y
7 Y
8 Y
9 Y
10 Y
11 Y
Maturity
28/02/2005
31/03/2005
29/04/2005
31/05/2005
30/06/2005
29/07/2005
31/08/2005
28/02/2006
31/03/2006
28/04/2006
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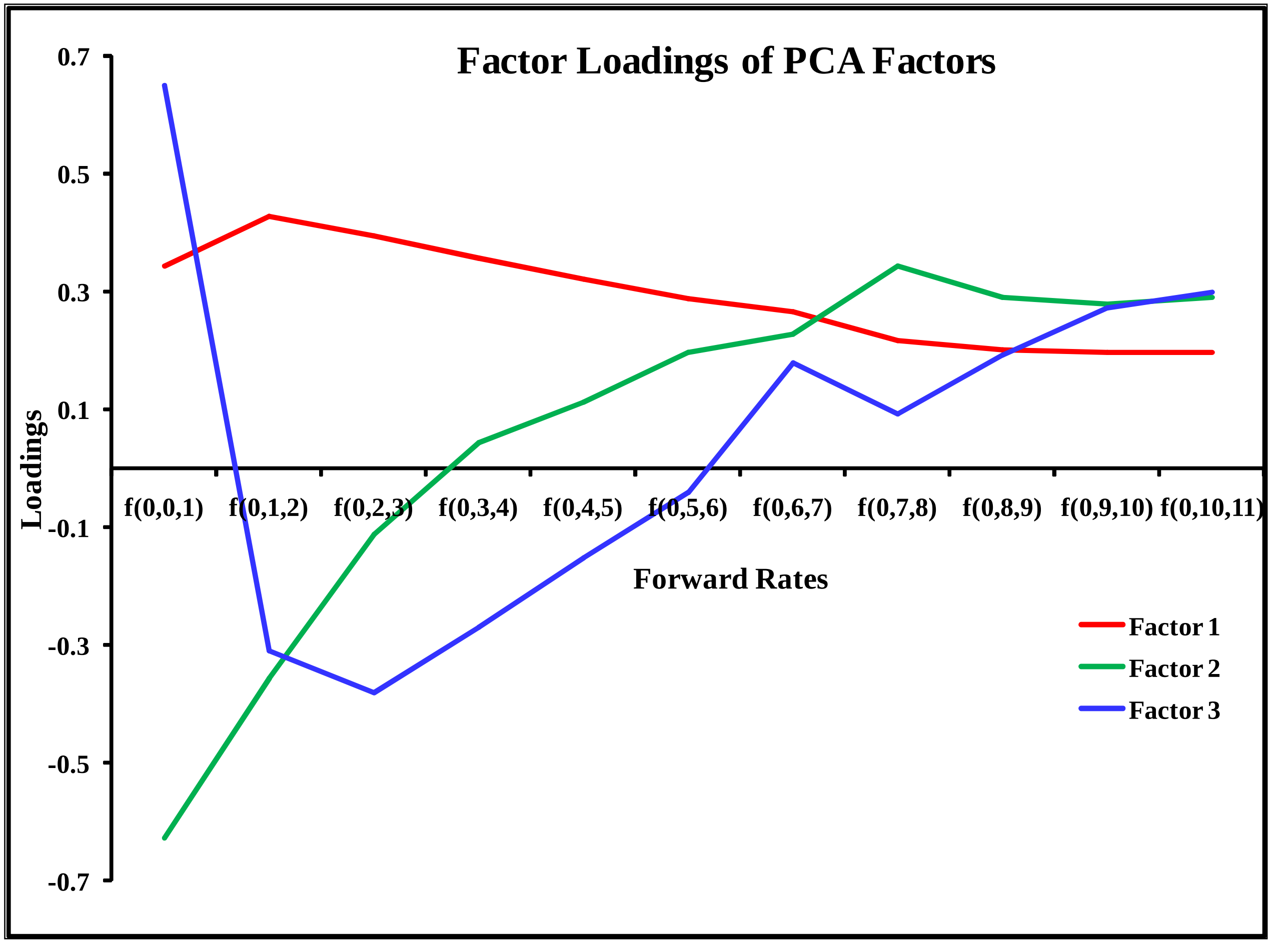
30/06/2006
31/07/2006
31/08/2006



Use NAG **g03aac** to carry out PCA analysis

Delta Hedging

- "Bumping" the yield curve by perturbing the first three PCA factors by $\pm$ mean absolute change.
- Assume no change in interest rate volatility.
- Calculate price sensitivities (delta) of 10x1 Bermudan swaption, and the hedge instruments (swaps, 1-, 5- and 11-year)
- Form a delta hedged portfolio at t i.e. Σ delta=0. Use NAG **e04ccc** (unconstrained min using simplex algorithm).
- Unwind at t+1 and use price information of 9x1 Bermudan swaption and swaps (0-, 4- and 10-year) to calculate portfolio value at t+1.
- Calculate P&L; good model should give P&L=0.

P&L Analysis

- nag_opt_simplex (e04ccc) is used
- RMSE of Hedging P&L

	HW	BK	SMM	LMM
EUR	1.486%	1.751%	1.886%	1.752%
USD	5.544%	5.484%	5.701%	7.754%

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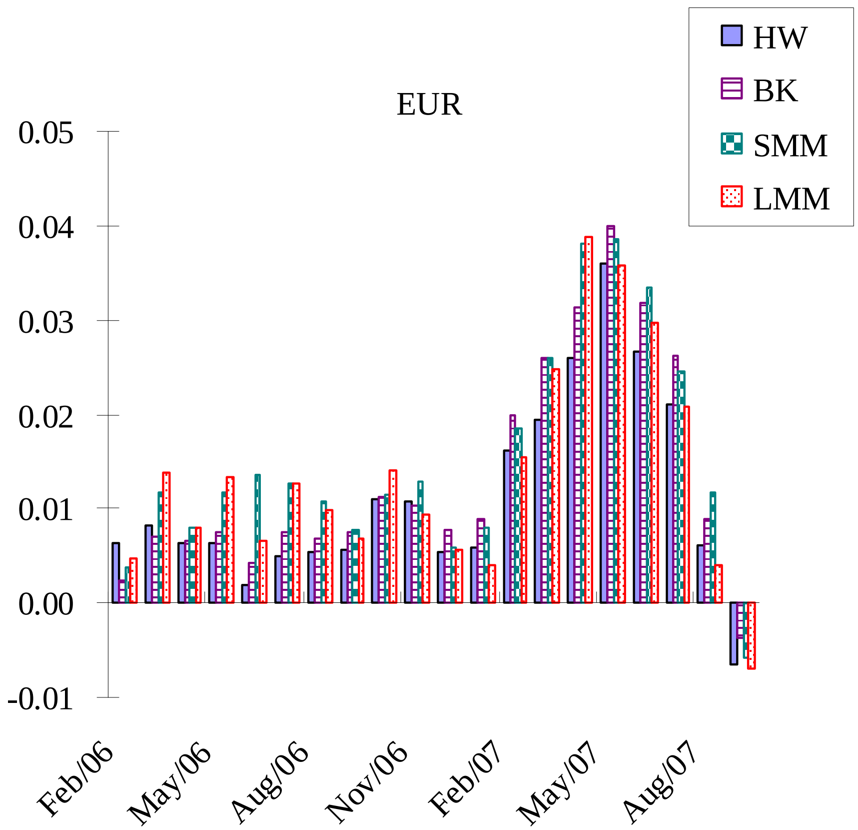
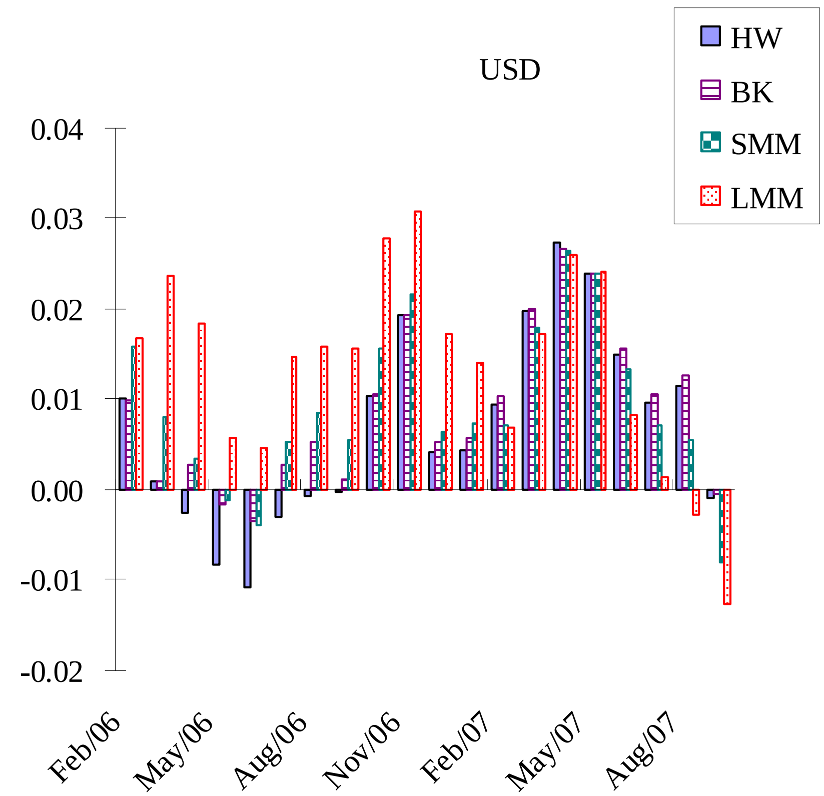
P&L of Hedging Results





Summary

- Pricing and hedging results are similar for all four models.
- HK~BK, and LMM~SMM
- For 1-factor pricing and long term risk management, there is little to choose between them. One should focus more on setting scenarios.
- Given 1-factor finding above, for ALM purpose, HW seems best; fast and better convergence property when compared with BK.

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NAG Visual Studio C++

- Visual C++ Express 2005 - CLDLL07XL (version 8 is now available)

<u>One time link for all current projects when you start working in the C++ studio</u>

- Started Visual Studio, click on Tools -> Options; Expand 'Projects and Solutions' and click on VC++ Directories;
- Change the 'Show directories for:' box so that it refers to include files and add the following line C:\Program Files\Numerical Algorithms Group\CLDLL074X\include
- Change the 'Show directories for:' box to show library files and add the following line C:\Program Files\Numerical Algorithms Group\CLDLL074X\include
- Finally change the box to show Executable files and add the line C:\Program Files\Numerical Algorithms Group\CLDLL074X\include
- Then click on OK. Do this once for all projects.

C++ (Cont'd)

To compile:

- Right click on the project name and click on 'properties'.
- Expand 'Configuration Properties' and then 'Linker'.
- Now click on 'Input'.
- The resulting window will contain an empty 'additional dependencies' line. Add nagc.lib to this. You should now be able to compile.

Calling C-Nag library from C++ code

- The best way to start is to download the example programme from NAG website, and modify the programme to suit your application.

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QFFE Industry Linked Project

- Recognition of the needs of financial practitioners. The course is structured to
  - 1) better prepare the students for life in market roles, and
  - 2) investigate real world issues through the dissertation.

- Invitation for small, self-contained industry relevant research topics
- Students are available for internship from September to December.
- Contributors include:
  - Alliance and Leicester,
  - Standard Bank
  - Morgan Stanley
  - ABN AMRO
  - Shell
  - Numerical Algorithms Group
  - Riskmetrics