

Using the NAG Library to solve a General Equilibrium Model with Endogenous Stock Market Non-participation

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A significant fraction of the population in any country with a liquid stock market do not hold stocks, despite the international evidence on a historically high equity risk premium. Whereas the phenomenon of Limited Stock Market Participation has been used to explain several other asset pricing puzzles, only a few studies address the equilibrium foundation for non-participation. Yet, to understand the sources of non-participation is of great importance for portfolio managers, regulators and academics. We develop a consumption based asset pricing model, in the spirit of Lucas (1978), with incomplete financial markets. The model incorporates heterogeneous endowments, heterogeneous labour income processes, and heterogeneous preferences with external additive habit formation, also known as “Catching up with the Joneses,” that is: investors maximize life-time utility over surplus consumption. One investor serves as the representative stand-in for all stockholders, whereas the second would-be investor represents potential non-participants.

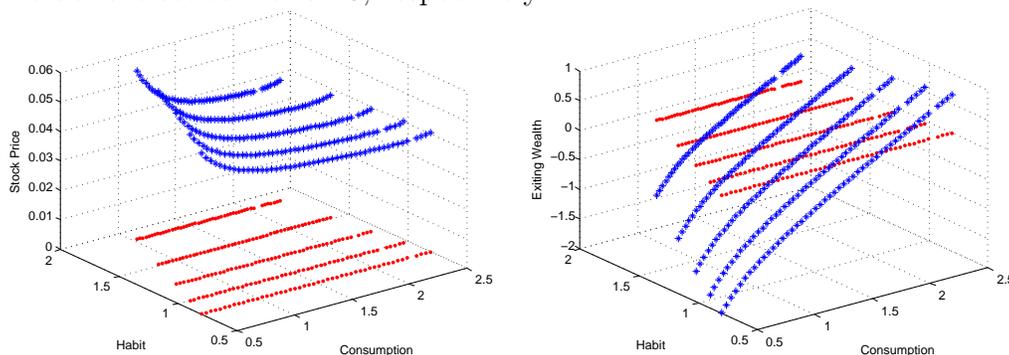
The main result of our research is that heterogeneity in endowments and preferences plus uninsurable labour income can deter one class of investors from entering the stock market. For this result, we do not need to introduce frictions such as fixed or variable costs to prevent one class of investors, which we identify as the poor, from participation. Using heterogeneous income risk allows us to match the consumption profiles of participants and non-participants in the data and heterogeneous external additive habit preferences allows us to reproduce many stylized facts of financial markets in our model.

We define our economy on a quadrinomial event tree (He (1990)) representing aggregate dividends and the labour income of participants and non-participants. To numerically solve our incomplete market economy, we use the recursive backward scheme described in Dumas and Lyasoff (2012). The scheme employs consumption instead of entering portfolio holdings as endogenous state variables, reducing the dimensionality of the grid, compared to the grid that is implied by pre-trade portfolios. In our model however, habit introduces path-dependency. To allow working on a recombining tree, we treat the habit as an additional endogenous state variable. Our code adjusts the Dumas and Lyasoff (2012) scheme to adapt for external additive habit with multi-period persistence and a short-sale constraint that models the uninsurability of income. The additive habit specification implies stationary wealth distributions and asset prices, hence extinction of investors cannot occur.

The multi-lag habit introduces a strong finite time effect. Therefore, to produce realistic and stationary asset pricing moments, we require at least 20 annual periods on our quadrinomial tree. On each node on the event tree, the endogenous state variables of the model, habit and consumption of the participant, form a non-rectangular two-dimensional grid. For each grid value, we call a solver to compute equilibrium given the particular grid point. This is a computationally intensive procedure, given a high number of grid points at each node on

the event tree and the high number of nodes given the time horizon of 20 years. Further, we interpolate the stock price and financial wealth, as visualized in Figure 1, over the non-rectangular grid and pass them over to the next backward step on the event tree. In this step, the solver evaluates values from interpolated functions, which is a very time-consuming procedure. Additionally, the same procedure is repeated for a broad number of parameter variations for the purpose of a sensitivity analysis and the sake of robustness.

Figure 1: The figures show the **stock price** and **financial wealth** (asterisks) of the stockholder against a **habit-consumption grid** (dots) for one particular node of the quadrinomial He (1990) tree with 20 annual periods. The parameters of relative risk aversion of stockholder and non-stockholder are set to 4 and 10, respectively.



For interpolation, we require a routine that allows us to interpolate over arbitrary distributed points on the plane. For this we rely on the NAG Library, provided by The Numerical Algorithms Group (www.nag.com). Interpolation routines are provided in the Chapter E01. This chapter contains a range of approaches that are suited to different problem types, depending on number of variables and other criteria. We use the fast and accurate implementation of the Renka and Cline (1984) routine, designated as `nag_interp_2d_scatt` (e01sa).

References

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