NAG Library Chapter Contents

S – Approximations of Special Functions

S Chapter Introduction

Routine Name	Mark of Introduction	Purpose
S01BAF	14	nagf_specfun_log_shifted $\ln(1 + x)$
S01EAF	14	nagf_specfun_exp_complex Complex exponential, e^z
S07AAF	1	nagf_specfun_tan $\tan x$
S09AAF	1	nagf_specfun_arcsin $\arcsin x$
S09ABF	3	nagf_specfun_arccos $\arccos x$
S10AAF	3	nagf_specfun_tanh $\tanh x$
S10ABF	4	nagf_specfun_sinh $\sinh x$
S10ACF	4	nagf_specfun_cosh $\cosh x$
S11AAF	4	nagf_specfun_arctanh $\operatorname{arctanh} x$
S11ABF	4	nagf_specfun_arcsinh $\operatorname{arcsinh} x$
S11ACF	4	nagf_specfun_arccosh $\operatorname{arccosh} x$
S13AAF	1	nagf_specfun_integral_exp Exponential integral $E_1(x)$
S13ACF	2	nagf_specfun_integral_cos Cosine integral $\operatorname{Ci}(x)$
S13ADF	5	nagf_specfun_integral_sin Sine integral $\operatorname{Si}(x)$
S14AAF	1	nagf_specfun_gamma Gamma function
S14ABF	8	nagf_specfun_gamma_log_real Log gamma function, real argument
S14ACF	14	nagf_specfun_polygamma $\psi(x) - \ln x$
S14ADF	14	nagf_specfun_polygamma_deriv Scaled derivatives of $\psi(x)$

S14AEF	20	nagf_specfun_psi_deriv_real
		Polygamma function $\psi^{(n)}(x)$ for real x
S14AFF	20	nagf_specfun_psi_deriv_complex
		Polygamma function $\psi^{(n)}(z)$ for complex z
S14AGF	21	nagf_specfun_gamma_log_complex
		Logarithm of the gamma function $\ln \Gamma(z)$, complex argument
S14AHF	23	nagf_specfun_gamma_log_scaled_real
		Scaled log gamma function
S14BAF	14	nagf_specfun_gamma_incomplete
		Incomplete gamma functions $P(a, x)$ and $Q(a, x)$
S14CBF	24	nagf_specfun_beta_log_real
		Logarithm of the beta function $\ln(B, a, b)$
S14CCF	24	nagf_specfun_beta_incomplete
		Incomplete beta function $I_x(a, b)$ and its complement $1 - I_x$
S15ABF	3	nagf_specfun_cdf_normal
		Cumulative Normal distribution function $P(x)$
S15ACF	4	nagf_specfun_compcdf_normal
		Complement of cumulative Normal distribution function $Q(x)$
S15ADF	4	nagf_specfun_erfc_real
		Complement of error function $\mathrm{erfc}(x)$
S15AEF	4	nagf_specfun_erf_real
		Error function $\mathrm{erf}(x)$
S15AFF	7	nagf_specfun_dawson
		Dawson's integral
S15AGF	22	nagf_specfun_erfcx_real
		Scaled complement of error function, $\mathrm{erfcx}(x)$
S15DDF	14	nagf_specfun_erfc_complex
		Scaled complex complement of error function, $\exp\left(-z^2\right)\mathrm{erfc}(-iz)$
S17ACF	1	nagf_specfun_bessel_y0_real
		Bessel function $Y_0(x)$
S17ADF	1	nagf_specfun_bessel_y1_real
		Bessel function $Y_1(x)$
S17AEF	5	nagf_specfun_bessel_j0_real
		Bessel function $J_0(x)$
S17AFF	5	nagf_specfun_bessel_j1_real
		Bessel function $J_1(x)$
S17AGF	8	nagf_specfun_airy_ai_real
		Airy function $\mathrm{Ai}(x)$
S17AHF	8	nagf_specfun_airy_bi_real
		Airy function $\mathrm{Bi}(x)$
S17AJF	8	nagf_specfun_airy_ai_deriv
		Airy function $\mathrm{Ai}'(x)$
S17AKF	8	nagf_specfun_airy_bi_deriv
		Airy function $\mathrm{Bi}'(x)$
S17ALF	20	nagf_specfun_bessel_zeros
		Zeros of Bessel functions $J_\alpha(x)$, $J_\alpha'(x)$, $Y_\alpha(x)$ or $Y_\alpha'(x)$

S17AQF	24	nagf_specfun_bessel_y0_real_vector Bessel function vectorized $Y_0(x)$
S17ARF	24	nagf_specfun_bessel_y1_real_vector Bessel function vectorized $Y_1(x)$
S17ASF	24	nagf_specfun_bessel_j0_real_vector Bessel function vectorized $J_0(x)$
S17ATF	24	nagf_specfun_bessel_j1_real_vector Bessel function vectorized $J_1(x)$
S17AUF	24	nagf_specfun_airy_ai_real_vector Airy function vectorized $\mathrm{Ai}(x)$
S17AVF	24	nagf_specfun_airy_bi_real_vector Airy function vectorized $\mathrm{Bi}(x)$
S17AWF	24	nagf_specfun_airy_ai_deriv_vector Airy function vectorized $\mathrm{Ai}'(x)$
S17AXF	24	nagf_specfun_airy_bi_deriv_vector Airy function vectorized $\mathrm{Bi}'(x)$
S17DCF	13	nagf_specfun_bessel_y_complex Bessel functions $Y_{\nu+a}(z)$, real $a \geq 0$, complex z, $\nu = 0, 1, 2, \ldots$
S17DEF	13	nagf_specfun_bessel_j_complex Bessel functions $J_{\nu+a}(z)$, real $a \geq 0$, complex z, $\nu = 0, 1, 2, \ldots$
S17DGF	13	nagf_specfun_airy_ai_complex Airy functions $\mathrm{Ai}(z)$ and $\mathrm{Ai}'(z)$, complex z
S17DHF	13	nagf_specfun_airy_bi_complex Airy functions $\mathrm{Bi}(z)$ and $\mathrm{Bi}'(z)$, complex z
S17DLF	13	nagf_specfun_hankel_complex Hankel functions $H_{\nu+a}^{(j)}(z)$, $j = 1, 2$, real $a \geq 0$, complex z, $\nu = 0, 1, 2, \ldots$
S18ACF	1	nagf_specfun_bessel_k0_real Modified Bessel function $K_0(x)$
S18ADF	1	nagf_specfun_bessel_k1_real Modified Bessel function $K_1(x)$
S18AEF	5	nagf_specfun_bessel_i0_real Modified Bessel function $I_0(x)$
S18AFF	5	nagf_specfun_bessel_i1_real Modified Bessel function $I_1(x)$
S18AQF	24	nagf_specfun_bessel_k0_real_vector Modified Bessel function vectorized $K_0(x)$
S18ARF	24	nagf_specfun_bessel_k1_real_vector Modified Bessel function vectorized $K_1(x)$
S18ASF	24	nagf_specfun_bessel_i0_real_vector Modified Bessel function vectorized $I_0(x)$
S18ATF	24	nagf_specfun_bessel_i1_real_vector Modified Bessel function vectorized $I_1(x)$
S18CCF	10	nagf_specfun_bessel_k0_scaled Scaled modified Bessel function $e^x K_0(x)$
S18CDF	10	nagf_specfun_bessel_k1_scaled Scaled modified Bessel function $e^x K_1(x)$

S18CEF	10	nagf_specfun_bessel_i0_scaled Scaled modified Bessel function $e^{-|x|}I_0(x)$
S18CFF	10	nagf_specfun_bessel_i1_scaled Scaled modified Bessel function $e^{-|x|}I_1(x)$
S18CQF	24	nagf_specfun_bessel_k0_scaled_vector Scaled modified Bessel function vectorized $e^x K_0(x)$
S18CRF	24	nagf_specfun_bessel_k1_scaled_vector Scaled modified Bessel function vectorized $e^x K_1(x)$
S18CSF	24	nagf_specfun_bessel_i0_scaled_vector Scaled modified Bessel function vectorized $e^{-|x|}I_0(x)$
S18CTF	24	nagf_specfun_bessel_i1_scaled_vector Scaled modified Bessel function vectorized $e^{-|x|}I_1(x)$
S18DCF	13	nagf_specfun_bessel_k_complex Modified Bessel functions $K_{\nu+a}(z)$, real $a \geq 0$, complex z, $\nu = 0, 1, 2, \ldots$
S18DEF	13	nagf_specfun_bessel_i_complex Modified Bessel functions $I_{\nu+a}(z)$, real $a \geq 0$, complex z, $\nu = 0, 1, 2, \ldots$
S18GKF	21	nagf_specfun_bessel_j_seq_complex Bessel function of the 1st kind $J_{\alpha \pm n}(z)$
S19AAF	11	nagf_specfun_kelvin_ber Kelvin function ber x
S19ABF	11	nagf_specfun_kelvin_bei Kelvin function bei x
S19ACF	11	nagf_specfun_kelvin_ker Kelvin function ker x
S19ADF	11	nagf_specfun_kelvin_kei Kelvin function kei x
S19ANF	24	nagf_specfun_kelvin_ber_vector Kelvin function vectorized ber x
S19APF	24	nagf_specfun_kelvin_bei_vector Kelvin function vectorized bei x
S19AQF	24	nagf_specfun_kelvin_ker_vector Kelvin function vectorized ker x
S19ARF	24	nagf_specfun_kelvin_kei_vector Kelvin function vectorized kei x
S20ACF	5	nagf_specfun_fresnel_s Fresnel integral $S(x)$
S20ADF	5	nagf_specfun_fresnel_c Fresnel integral $C(x)$
S20AQF	24	nagf_specfun_fresnel_s_vector Fresnel integral vectorized $S(x)$
S20ARF	24	nagf_specfun_fresnel_c_vector Fresnel integral vectorized $C(x)$
S21BAF	8	nagf_specfun_ellipint_symm_1_degen Degenerate symmetrised elliptic integral of 1st kind $R_C(x, y)$
S21BBF	8	nagf_specfun_ellipint_symm_1 Symmetrised elliptic integral of 1st kind $R_F(x, y, z)$

S21BCF	8	nagf_specfun_ellipint_symm_2 Symmetrised elliptic integral of 2nd kind $R_D(x, y, z)$
S21BDF	8	nagf_specfun_ellipint_symm_3 Symmetrised elliptic integral of 3rd kind $R_J(x, y, z, r)$
S21BEF	22	nagf_specfun_ellipint_legendre_1 Elliptic integral of 1st kind, Legendre form, $F(\phi \mid m)$
S21BFF	22	nagf_specfun_ellipint_legendre_2 Elliptic integral of 2nd kind, Legendre form, $E(\phi \mid m)$
S21BGF	22	nagf_specfun_ellipint_legendre_3 Elliptic integral of 3rd kind, Legendre form, $\Pi(n; \phi \mid m)$
S21BHF	22	nagf_specfun_ellipint_complete_1 Complete elliptic integral of 1st kind, Legendre form, $K(m)$
S21BJF	22	nagf_specfun_ellipint_complete_2 Complete elliptic integral of 2nd kind, Legendre form, $E(m)$
S21CAF	15	nagf_specfun_jacellip_real Jacobian elliptic functions sn, cn and dn of real argument
S21CBF	20	nagf_specfun_jacellip_complex Jacobian elliptic functions sn, cn and dn of complex argument
S21CCF	20	nagf_specfun_jactheta_real Jacobian theta functions $\theta_k(x, q)$ of real argument
S21DAF	20	nagf_specfun_ellipint_general_2 General elliptic integral of 2nd kind $F(z, k', a, b)$ of complex argument
S22AAF	20	nagf_specfun_legendre_p Legendre functions of 1st kind $P_n^m(x)$ or $\overline{P_n^m}(x)$
S22BAF	24	nagf_specfun_1f1_real Real confluent hypergeometric function ${}_1F_1(a; b; x)$
S22BBF	24	nagf_specfun_1f1_real_scaled Real confluent hypergeometric function ${}_1F_1(a; b; x)$ in scaled form
S30AAF	22	nagf_specfun_opt_bsm_price Black–Scholes–Merton option pricing formula
S30ABF	22	nagf_specfun_opt_bsm_greeks Black–Scholes–Merton option pricing formula with Greeks
S30BAF	22	nagf_specfun_opt_lookback_fls_price Floating-strike lookback option pricing formula
S30BBF	22	nagf_specfun_opt_lookback_fls_greeks Floating-strike lookback option pricing formula with Greeks
S30CAF	22	nagf_specfun_opt_binary_con_price Binary option, cash-or-nothing pricing formula
S30CBF	22	nagf_specfun_opt_binary_con_greeks Binary option, cash-or-nothing pricing formula with Greeks
S30CCF	22	nagf_specfun_opt_binary_aon_price Binary option, asset-or-nothing pricing formula
S30CDF	22	nagf_specfun_opt_binary_aon_greeks Binary option, asset-or-nothing pricing formula with Greeks
S30FAF	22	nagf_specfun_opt_barrier_std_price Standard barrier option pricing formula

S30JAF	22	nagf_specfun_opt_jumpdiff_merton_price Jump-diffusion, Merton's model, option pricing formula
S30JBF	22	nagf_specfun_opt_jumpdiff_merton_greeks Jump-diffusion, Merton's model, option pricing formula with Greeks
S30NAF	22	nagf_specfun_opt_heston_price Heston's model option pricing formula
S30NBF	23	nagf_specfun_opt_heston_greeks Heston's model option pricing formula with Greeks
S30QCF	22	nagf_specfun_opt_amer_bs_price American option, Bjerksund and Stensland pricing formula
S30SAF	22	nagf_specfun_opt_asian_geom_price Asian option, geometric continuous average rate pricing formula
S30SBF	22	nagf_specfun_opt_asian_geom_greeks Asian option, geometric continuous average rate pricing formula with Greeks