NAG Library Routine Document

F07JNF (ZPTSV)

Note: before using this routine, please read the Users' Note for your implementation to check the interpretation of ***bold italicised*** terms and other implementation-dependent details.

1 Purpose

F07JNF (ZPTSV) computes the solution to a complex system of linear equations

$$AX = B,$$

where A is an n by n Hermitian positive definite tridiagonal matrix, and X and B are n by r matrices.

2 Specification

```
SUBROUTINE F07JNF (N, NRHS, D, E, B, LDB, INFO)

INTEGER                 N, NRHS, LDB, INFO
REAL (KIND=nag_wp)      D(*)
COMPLEX (KIND=nag_wp) E(*), B(LDB,*)
```

The routine may be called by its LAPACK name ***zptsv***.

3 Description

F07JNF (ZPTSV) factors A as $A = LDL^{\mathrm{H}}$. The factored form of A is then used to solve the system of equations.

4 References

Anderson E, Bai Z, Bischof C, Blackford S, Demmel J, Dongarra J J, Du Croz J J, Greenbaum A, Hammarling S, McKenney A and Sorensen D (1999) *LAPACK Users' Guide* (3rd Edition) SIAM, Philadelphia http://www.netlib.org/lapack/lug

Golub G H and Van Loan C F (1996) *Matrix Computations* (3rd Edition) Johns Hopkins University Press, Baltimore

5 Parameters

1: N – INTEGER *Input*

On entry: n, the order of the matrix A.

Constraint: $\mathrm{N} \geq 0$.

2: NRHS – INTEGER *Input*

On entry: r, the number of right-hand sides, i.e., the number of columns of the matrix B.

Constraint: $\mathrm{NRHS} \geq 0$.

3: D($*$) – REAL (KIND=nag_wp) array *Input/Output*

Note: the dimension of the array D must be at least $\max(1, \mathrm{N})$.

On entry: the n diagonal elements of the tridiagonal matrix A.

On exit: the n diagonal elements of the diagonal matrix D from the factorization $A = LDL^{\mathrm{H}}$.

4: E(∗) – COMPLEX (KIND=nag_wp) array *Input/Output*

Note: the dimension of the array E must be at least $\max(1, N - 1)$.

On entry: the $(n - 1)$ subdiagonal elements of the tridiagonal matrix A.

On exit: the $(n - 1)$ subdiagonal elements of the unit bidiagonal factor L from the LDL^{H} factorization of A. (E can also be regarded as the superdiagonal of the unit bidiagonal factor U from the $U^{H}DU$ factorization of A.)

5: B(LDB,∗) – COMPLEX (KIND=nag_wp) array *Input/Output*

Note: the second dimension of the array B must be at least $\max(1, NRHS)$.

On entry: the n by r right-hand side matrix B.

On exit: if INFO $= 0$, the n by r solution matrix X.

6: LDB – INTEGER *Input*

On entry: the first dimension of the array B as declared in the (sub)program from which F07JNF (ZPTSV) is called.

Constraint: LDB $\geq \max(1, N)$.

7: INFO – INTEGER *Output*

On exit: INFO $= 0$ unless the routine detects an error (see Section 6).

6 Error Indicators and Warnings

Errors or warnings detected by the routine:

INFO < 0

If INFO $= -i$, the ith argument had an illegal value. An explanatory message is output, and execution of the program is terminated.

INFO > 0

If INFO $= i$, the leading minor of order i is not positive definite, and the solution has not been computed. The factorization has not been completed unless $i = N$.

7 Accuracy

The computed solution for a single right-hand side, $\hat{x}$, satisfies an equation of the form

$$(A + E)\hat{x} = b,$$

where

$$\|E\|_1 = O(\epsilon)\|A\|_1$$

and ϵ is the ***machine precision***. An approximate error bound for the computed solution is given by

$$\frac{\|\hat{x} - x\|_1}{\|x\|_1} \leq \kappa(A)\frac{\|E\|_1}{\|A\|_1},$$

where $\kappa(A) = \|A^{-1}\|_1\|A\|_1$, the condition number of A with respect to the solution of the linear equations. See Section 4.4 of Anderson *et al.* (1999) for further details.

F07JPF (ZPTSVX) is a comprehensive LAPACK driver that returns forward and backward error bounds and an estimate of the condition number. Alternatively, F04CGF solves $Ax = b$ and returns a forward error bound and condition estimate. F04CGF calls F07JNF (ZPTSV) to solve the equations.

8 Further Comments

The number of floating point operations required for the factorization of A is proportional to n, and the number of floating point operations required for the solution of the equations is proportional to nr, where r is the number of right-hand sides.

The real analogue of this routine is F07JAF (DPTSV).

9 Example

This example solves the equations

$$Ax = b,$$

where A is the Hermitian positive definite tridiagonal matrix

$$A = \begin{pmatrix} 16.0 & 16.0 - 16.0i & 0 & 0 \\ 16.0 + 16.0i & 41.0 & 18.0 + 9.0i & 0 \\ 0 & 18.0 - 9.0i & 46.0 & 1.0 + 4.0i \\ 0 & 0 & 1.0 - 4.0i & 21.0 \end{pmatrix}$$

and

$$b = \begin{pmatrix} 64.0 + 16.0i \\ 93.0 + 62.0i \\ 78.0 - 80.0i \\ 14.0 - 27.0i \end{pmatrix}.$$

Details of the LDL^{H} factorization of A are also output.

9.1 Program Text

```
    Program f07jnfe

!     F07JNF Example Program Text

!     Mark 24 Release. NAG Copyright 2012.

!     .. Use Statements ..
      Use nag_library, Only: nag_wp, zptsv
!     .. Implicit None Statement ..
      Implicit None
!     .. Parameters ..
      Integer, Parameter               :: nin = 5, nout = 6
!     .. Local Scalars ..
      Integer                          :: info, n
!     .. Local Arrays ..
      Complex (Kind=nag_wp), Allocatable :: b(:), e(:)
      Real (Kind=nag_wp), Allocatable  :: d(:)
!     .. Executable Statements ..
      Write (nout,*) 'F07JNF Example Program Results'
      Write (nout,*)
!     Skip heading in data file
      Read (nin,*)
      Read (nin,*) n

      Allocate (b(n),e(n-1),d(n))

!     Read the lower bidiagonal part of the tridiagonal matrix A and
!     the right hand side b from data file

      Read (nin,*) d(1:n)
      Read (nin,*) e(1:n-1)
      Read (nin,*) b(1:n)

!     Solve the equations Ax = b for x
!     The NAG name equivalent of zptsv is f07jnf
```

```
      Call zptsv(n,1,d,e,b,n,info)

      If (info==0) Then

!       Print solution

        Write (nout,*) 'Solution'
        Write (nout,99999) b(1:n)

!       Print details of factorization

        Write (nout,*)
        Write (nout,*) 'Diagonal elements of the diagonal matrix D'
        Write (nout,99998) d(1:n)
        Write (nout,*)
        Write (nout,*) 'Sub-diagonal elements of the Cholesky factor L'
        Write (nout,99999) e(1:n-1)

      Else
        Write (nout,99997) 'The leading minor of order ', info, &
          ' is not positive definite'
      End If

99999 Format (4(' (',F8.4,',',F8.4,')':))
99998 Format ((2X,F7.4,3(11X,F7.4)))
99997 Format (1X,A,I3,A)
    End Program f07jnfe
```

9.2 Program Data

```
F07JNF Example Program Data
    4                                                      :Value of N
   16.0          41.0          46.0          21.0          :End of diagonal D
 ( 16.0, 16.0) ( 18.0, -9.0) (  1.0, -4.0)                 :End of sub-diagonal E
 ( 64.0, 16.0) ( 93.0, 62.0) ( 78.0,-80.0) ( 14.0,-27.0) :End of vector b
```

9.3 Program Results

```
 F07JNF Example Program Results

 Solution
 (   2.0000,  1.0000) (   1.0000,  1.0000) (   1.0000, -2.0000) (   1.0000, -1.0000)

 Diagonal elements of the diagonal matrix D
  16.0000            9.0000            1.0000            4.0000

 Sub-diagonal elements of the Cholesky factor L
 (   1.0000,  1.0000) (   2.0000, -1.0000) (   1.0000, -4.0000)
```