NAG Library Function Document

nag_quad_1d_gauss_wset (d01tbc)

1 Purpose

nag_quad_1d_gauss_wset (d01tbc) returns the weights and abscissae appropriate to a Gaussian quadrature formula with a specified number of abscissae. The formulae provided are for Gauss–Legendre, rational Gauss, Gauss–Laguerre and Gauss–Hermite.

2 Specification

```
#include <nag.h>
#include <nagd01.h>
void nag_quad_1d_gauss_wset (Nag_QuadType quad_type, double a, double b,
     Integer n, double weight[], double abscis[], NagError *fail)
```

3 Description

nag_quad_1d_gauss_wset (d01tbc) returns the weights and abscissae for use in the Gaussian quadrature of a function $f(x)$. The quadrature takes the form

$$S = \sum_{i=1}^{n} w_i f(x_i)$$

where w_i are the weights and x_i are the abscissae (see Davis and Rabinowitz (1975), Fröberg (1970), Ralston (1965) or Stroud and Secrest (1966)).

Weights and abscissae are available for Gauss–Legendre, rational Gauss, Gauss–Laguerre and Gauss–Hermite quadrature, and for a selection of values of n (see Section 5).

(a) Gauss–Legendre Quadrature:

$$S \simeq \int_a^b f(x)\,dx$$

where a and b are finite and it will be exact for any function of the form

$$f(x) = \sum_{i=0}^{2n-1} c_i x^i.$$

(b) Rational Gauss quadrature, adjusted weights:

$$S \simeq \int_a^\infty f(x)\,dx \quad (a+b>0) \quad \text{or} \quad S \simeq \int_{-\infty}^a f(x)\,dx \quad (a+b<0)$$

and will be exact for any function of the form

$$f(x) = \sum_{i=2}^{2n+1} \frac{c_i}{(x+b)^i} = \frac{\sum_{i=0}^{2n-1} c_{2n+1-i}(x+b)^i}{(x+b)^{2n+1}}.$$

(c) Gauss–Laguerre quadrature, adjusted weights:

$$S \simeq \int_a^\infty f(x)\,dx \quad (b>0) \quad \text{or} \quad S \simeq \int_{-\infty}^a f(x)\,dx \quad (b<0)$$

and will be exact for any function of the form

$$f(x) = e^{-bx}\sum_{i=0}^{2n-1} c_i x^i.$$

(d) Gauss–Hermite quadrature, adjusted weights:

$$S \simeq \int_{-\infty}^{+\infty} f(x)\,dx$$

and will be exact for any function of the form

$$f(x) = e^{-b(x-a)^2}\sum_{i=0}^{2n-1} c_i x^i \quad (b > 0).$$

(e) Gauss–Laguerre quadrature, normal weights:

$$S \simeq \int_a^{\infty} e^{-bx} f(x)\,dx \quad (b > 0) \quad \text{or} \quad S \simeq \int_{-\infty}^{a} e^{-bx} f(x)\,dx \quad (b < 0)$$

and will be exact for any function of the form

$$f(x) = \sum_{i=0}^{2n-1} c_i x^i.$$

(f) Gauss–Hermite quadrature, normal weights:

$$S \simeq \int_{-\infty}^{+\infty} e^{-b(x-a)^2} f(x)\,dx$$

and will be exact for any function of the form

$$f(x) = \sum_{i=0}^{2n-1} c_i x^i.$$

Note: the Gauss–Legendre abscissae, with $a = -1$, $b = +1$, are the zeros of the Legendre polynomials; the Gauss–Laguerre abscissae, with $a = 0$, $b = 1$, are the zeros of the Laguerre polynomials; and the Gauss–Hermite abscissae, with $a = 0$, $b = 1$, are the zeros of the Hermite polynomials.

4 References

Davis P J and Rabinowitz P (1975) *Methods of Numerical Integration* Academic Press

Fröberg C E (1970) *Introduction to Numerical Analysis* Addison–Wesley

Ralston A (1965) *A First Course in Numerical Analysis* pp. 87–90 McGraw–Hill

Stroud A H and Secrest D (1966) *Gaussian Quadrature Formulas* Prentice–Hall

5 Arguments

1: **quad_type** – Nag_QuadType *Input*

On entry: indicates the quadrature formula.

quad_type = Nag_Quad_Gauss_Legendre
Gauss–Legendre quadrature on a finite interval, using normal weights.

quad_type = Nag_Quad_Gauss_Laguerre
Gauss–Laguerre quadrature on a semi-infinite interval, using normal weights.

quad_type = Nag_Quad_Gauss_Laguerre_Adjusted
Gauss–Laguerre quadrature on a semi-infinite interval, using adjusted weights.

quad_type = Nag_Quad_Gauss_Hermite
Gauss–Hermite quadrature on an infinite interval, using normal weights.

quad_type = Nag_Quad_Gauss_Hermite_Adjusted
Gauss–Hermite quadrature on an infinite interval, using adjusted weights.

quad_type = Nag_Quad_Gauss_Rational_Adjusted
Rational Gauss quadrature on a semi-infinite interval, using adjusted weights.

Constraint: **quad_type** = Nag_Quad_Gauss_Legendre, Nag_Quad_Gauss_Laguerre, Nag_Quad_Gauss_Laguerre_Adjusted, Nag_Quad_Gauss_Hermite, Nag_Quad_Gauss_Hermite_Adjusted or Nag_Quad_Gauss_Rational_Adjusted.

2: **a** – double *Input*

3: **b** – double *Input*

On entry: the quantities a and b as described in the appropriate sub-section of Section 3.

Constraints:

if **quad_type** = Nag_Quad_Gauss_Rational_Adjusted, $\mathbf{a} + \mathbf{b} \neq 0.0$;
if **quad_type** = Nag_Quad_Gauss_Laguerre or Nag_Quad_Gauss_Laguerre_Adjusted, $\mathbf{b} \neq 0.0$;
if **quad_type** = Nag_Quad_Gauss_Hermite or Nag_Quad_Gauss_Hermite_Adjusted, $\mathbf{b} > 0.0$.

Constraints:

Rational Gauss: $\mathbf{a} + \mathbf{b} \neq 0.0$;
Gauss–Laguerre: $\mathbf{b} \neq 0.0$;
Gauss–Hermite: $\mathbf{b} > 0$.

4: **n** – Integer *Input*

On entry: n, the number of weights and abscissae to be returned.

Constraint: **n** = 1, 2, 3, 4, 5, 6, 8, 10, 12, 14, 16, 20, 24, 32, 48 or 64.

Note: if $n > 0$ and is not a member of the above list, the maxmium value of n stored below n will be used, and all subsequent elements of **abscis** and **weight** will be returned as zero.

5: **weight**[**n**] – double *Output*

On exit: the **n** weights.

6: **abscis**[**n**] – double *Output*

On exit: the **n** abscissae.

7: **fail** – NagError * *Input/Output*

The NAG error argument (see Section 3.6 in the Essential Introduction).

6 Error Indicators and Warnings

NE_BAD_PARAM

The value of **a** and/or **b** is invalid for the chosen **quad_type**. Either:

On entry, argument ⟨*value*⟩ had an illegal value.

The value of **a** and/or **b** is invalid for Gauss-Hermite quadrature.
On entry, **quad_type** = ⟨*value*⟩.
On entry, **a** = ⟨*value*⟩ and **b** = ⟨*value*⟩.
Constraint: **b** > 0.0.

The value of **a** and/or **b** is invalid for Gauss-Laguerre quadrature.
On entry, **quad_type** = ⟨*value*⟩.
On entry, **a** = ⟨*value*⟩ and **b** = ⟨*value*⟩.
Constraint: $|\mathbf{b}| > 0.0$.

The value of **a** and/or **b** is invalid for rational Gauss quadrature.
On entry, **quad_type** = ⟨*value*⟩.
On entry, **a** = ⟨*value*⟩ and **b** = ⟨*value*⟩.
Constraint: $|\mathbf{a} + \mathbf{b}| > 0.0$.

NE_INT

On entry, **n** = ⟨*value*⟩.
Constraint: **n** > 0.

NE_INTERNAL_ERROR

An internal error has occurred in this function. Check the function call and any array sizes. If the call is correct then please contact NAG for assistance.

NE_QUAD_GAUSS_NPTS_RULE

The **n**-point rule is not among those stored.
On entry: **n** = ⟨*value*⟩.
n-rule used: **n** = ⟨*value*⟩.

NE_TOO_SMALL

Underflow occurred in calculation of normal weights.
Reduce **n** or use adjusted weights: **n** = ⟨*value*⟩.

NE_WEIGHT_ZERO

No nonzero weights were generated for the provided parameters.

7 Accuracy

The weights and abscissae are stored for standard values of **a** and **b** to full machine accuracy.

8 Parallelism and Performance

Not applicable.

9 Further Comments

Timing is negligible.

10 Example

This example returns the abscissae and (adjusted) weights for the six-point Gauss–Laguerre formula.

10.1 Program Text

```
/* nag_quad_1d_gauss_wset (d01tbc) Example Program.
 *
 * Copyright 2011, Numerical Algorithms Group.
 *
 * Mark 23, 2011.
 */

#include <stdio.h>
#include <nag.h>
```

```
#include <nag_stdlib.h>
#include <nagd01.h>

int main(void)
{
  Integer exit_status = 0;
  Integer n;
  double  a, b;
  Integer i;
  Nag_QuadType quadtype;
  NagError fail;
  double   *abscis = 0, *weight = 0;

  INIT_FAIL(fail);

  printf("nag_quad_1d_gauss_wset (d01tbc) Example Program Results\n");
  /* Skip heading in data file */
  scanf("%*[^\n] ");
  /* Input a, b and n */
  scanf("%lf %lf", &a, &b);
  scanf("%ld%*[^\n] ", &n);
  quadtype = Nag_Quad_Gauss_Laguerre_Adjusted;

  if (!(abscis = NAG_ALLOC(n, double)) ||
      !(weight = NAG_ALLOC(n, double)))
    {
      printf("Allocation failure\n");
      exit_status = -1;
      goto END;
    }

  /* nag_quad_1d_gauss_wset (d01tbc).
   * Pre-computed weights and abscissae for
   * Gaussian quadrature rules, restricted choice of rule.
   */
  nag_quad_1d_gauss_wset(quadtype, a, b, n, weight, abscis, &fail);
  if (fail.code != NE_NOERROR)
    {
      printf("Error from nag_quad_1d_gauss_wset (d01tbc).\n%s\n",
              fail.message);
      exit_status = 1;
      goto END;
    }

  printf("\nLaguerre formula, %3ld points\n\n"
         "    Abscissae        Weights\n\n", n);
  for (i = 0; i < n; i++)
    {
      printf("%15.6e", abscis[i]);
      printf("%15.6e\n", weight[i]);
    }
  printf("\n");

 END:
  NAG_FREE(abscis);
  NAG_FREE(weight);

  return exit_status;
}
```

10.2 Program Data

None.

10.3 Program Results

```
nag_quad_1d_gauss_wset (d01tbc) Example Program Results

Laguerre formula,   6 points
```

```
    Abscissae         Weights

   2.228466e-01   5.735355e-01
   1.188932e+00   1.369253e+00
   2.992736e+00   2.260685e+00
   5.775144e+00   3.350525e+00
   9.837467e+00   4.886827e+00
   1.598287e+01   7.849016e+00
```