

# NAG Library Function Document

## nag\_zhecon (f07muc)

### 1 Purpose

nag\_zhecon (f07muc) estimates the condition number of a complex Hermitian indefinite matrix  $A$ , where  $A$  has been factorized by nag\_zhetrf (f07mrc).

### 2 Specification

```
#include <nag.h>
#include <nagf07.h>

void nag_zhecon (Nag_OrderType order, Nag_UploType uplo, Integer n,
                 const Complex a[], Integer pda, const Integer ipiv[], double anorm,
                 double *rcond, NagError *fail)
```

### 3 Description

nag\_zhecon (f07muc) estimates the condition number (in the 1-norm) of a complex Hermitian indefinite matrix  $A$ :

$$\kappa_1(A) = \|A\|_1 \|A^{-1}\|_1.$$

Since  $A$  is Hermitian,  $\kappa_1(A) = \kappa_\infty(A) = \|A\|_\infty \|A^{-1}\|_\infty$ .

Because  $\kappa_1(A)$  is infinite if  $A$  is singular, the function actually returns an estimate of the **reciprocal** of  $\kappa_1(A)$ .

The function should be preceded by a call to nag\_zhe\_norm (f16ucc) to compute  $\|A\|_1$  and a call to nag\_zhetrf (f07mrc) to compute the Bunch–Kaufman factorization of  $A$ . The function then uses Higham’s implementation of Hager’s method (see Higham (1988)) to estimate  $\|A^{-1}\|_1$ .

### 4 References

Higham N J (1988) FORTRAN codes for estimating the one-norm of a real or complex matrix, with applications to condition estimation *ACM Trans. Math. Software* **14** 381–396

### 5 Arguments

1: **order** – Nag\_OrderType *Input*

*On entry:* the **order** argument specifies the two-dimensional storage scheme being used, i.e., row-major ordering or column-major ordering. C language defined storage is specified by **order** = Nag\_RowMajor. See Section 3.2.1.3 in the Essential Introduction for a more detailed explanation of the use of this argument.

*Constraint:* **order** = Nag\_RowMajor or Nag\_ColMajor.

2: **uplo** – Nag\_UploType *Input*

*On entry:* specifies how  $A$  has been factorized.

**uplo** = Nag\_Upper

$A = PUDU^H P^T$ , where  $U$  is upper triangular.

**uplo** = Nag\_Lower  
 $A = PLDL^H P^T$ , where  $L$  is lower triangular.

*Constraint:* **uplo** = Nag\_Upper or Nag\_Lower.

- 3: **n** – Integer *Input*  
*On entry:*  $n$ , the order of the matrix  $A$ .  
*Constraint:*  $n \geq 0$ .
- 4: **a**[ $dim$ ] – const Complex *Input*  
**Note:** the dimension,  $dim$ , of the array **a** must be at least  $\max(1, pda \times n)$ .  
*On entry:* details of the factorization of  $A$ , as returned by nag\_zhetrf (f07mrc).
- 5: **pda** – Integer *Input*  
*On entry:* the stride separating row or column elements (depending on the value of **order**) of the matrix in the array **a**.  
*Constraint:*  $pda \geq \max(1, n)$ .
- 6: **ipiv**[ $dim$ ] – const Integer *Input*  
**Note:** the dimension,  $dim$ , of the array **ipiv** must be at least  $\max(1, n)$ .  
*On entry:* details of the interchanges and the block structure of  $D$ , as returned by nag\_zhetrf (f07mrc).
- 7: **anorm** – double *Input*  
*On entry:* the 1-norm of the **original** matrix  $A$ , which may be computed by calling nag\_zhe\_norm (f16ucc) with its argument **norm** = Nag\_OneNorm. **anorm** must be computed either **before** calling nag\_zhetrf (f07mrc) or else from a **copy** of the original matrix  $A$ .  
*Constraint:*  $anorm \geq 0.0$ .
- 8: **rcond** – double \* *Output*  
*On exit:* an estimate of the reciprocal of the condition number of  $A$ . **rcond** is set to zero if exact singularity is detected or the estimate underflows. If **rcond** is less than *machine precision*,  $A$  is singular to working precision.
- 9: **fail** – NagError \* *Input/Output*  
The NAG error argument (see Section 3.6 in the Essential Introduction).

## 6 Error Indicators and Warnings

### NE\_ALLOC\_FAIL

Dynamic memory allocation failed.

### NE\_BAD\_PARAM

On entry, argument  $\langle value \rangle$  had an illegal value.

### NE\_INT

On entry, **n** =  $\langle value \rangle$ .

Constraint:  $n \geq 0$ .

On entry, **pda** =  $\langle value \rangle$ .

Constraint:  $pda > 0$ .

**NE\_INT\_2**

On entry, **pda** =  $\langle value \rangle$  and **n** =  $\langle value \rangle$ .  
 Constraint: **pda**  $\geq$   $\max(1, \mathbf{n})$ .

**NE\_INTERNAL\_ERROR**

An internal error has occurred in this function. Check the function call and any array sizes. If the call is correct then please contact NAG for assistance.

**NE\_REAL**

On entry, **anorm** =  $\langle value \rangle$ .  
 Constraint: **anorm**  $\geq$  0.0.

**7 Accuracy**

The computed estimate **rcond** is never less than the true value  $\rho$ , and in practice is nearly always less than  $10\rho$ , although examples can be constructed where **rcond** is much larger.

**8 Parallelism and Performance**

nag\_zhecon (f07muc) is not threaded by NAG in any implementation.

nag\_zhecon (f07muc) makes calls to BLAS and/or LAPACK routines, which may be threaded within the vendor library used by this implementation. Consult the documentation for the vendor library for further information.

Please consult the Users' Note for your implementation for any additional implementation-specific information.

**9 Further Comments**

A call to nag\_zhecon (f07muc) involves solving a number of systems of linear equations of the form  $Ax = b$ ; the number is usually 5 and never more than 11. Each solution involves approximately  $8n^2$  real floating-point operations but takes considerably longer than a call to nag\_zhetrs (f07muc) with one right-hand side, because extra care is taken to avoid overflow when  $A$  is approximately singular.

The real analogue of this function is nag\_dsycon (f07mgc).

**10 Example**

This example estimates the condition number in the 1-norm (or  $\infty$ -norm) of the matrix  $A$ , where

$$A = \begin{pmatrix} -1.36 + 0.00i & 1.58 + 0.90i & 2.21 - 0.21i & 3.91 + 1.50i \\ 1.58 - 0.90i & -8.87 + 0.00i & -1.84 - 0.03i & -1.78 + 1.18i \\ 2.21 + 0.21i & -1.84 + 0.03i & -4.63 + 0.00i & 0.11 + 0.11i \\ 3.91 - 1.50i & -1.78 - 1.18i & 0.11 - 0.11i & -1.84 + 0.00i \end{pmatrix}.$$

Here  $A$  is Hermitian indefinite and must first be factorized by nag\_zhetrf (f07mrc). The true condition number in the 1-norm is 9.10.

**10.1 Program Text**

```
/* nag_zhecon (f07muc) Example Program.
 *
 * Copyright 2001 Numerical Algorithms Group.
 *
 * Mark 7, 2001.
 */

#include <stdio.h>
#include <nag.h>
```

```

#include <nag_stdlib.h>
#include <naga02.h>
#include <nagf07.h>
#include <nagf16.h>
#include <nagx02.h>

int main(void)
{
    /* Scalars */
    double      anorm, rcond;
    Integer     i, j, n, pda;
    Integer     exit_status = 0;
    Nag_UploType uplo;
    NagError    fail;
    Nag_OrderType order;
    /* Arrays */
    Integer     *ipiv = 0;
    char        nag_enum_arg[40];
    Complex     *a = 0;

#ifdef NAG_COLUMN_MAJOR
#define A(I, J) a[(J-1)*pda + I - 1]
    order = Nag_ColMajor;
#else
#define A(I, J) a[(I-1)*pda + J - 1]
    order = Nag_RowMajor;
#endif

    INIT_FAIL(fail);

    printf("nag_zhecon (f07muc) Example Program Results\n\n");

    /* Skip heading in data file */
    scanf("%*[\n] ");
    scanf("%ld%*[\n] ", &n);
#ifdef NAG_COLUMN_MAJOR
    pda = n;
#else
    pda = n;
#endif

    /* Allocate memory */
    if (!(ipiv = NAG_ALLOC(n, Integer)) ||
        !(a = NAG_ALLOC(n * n, Complex)))
    {
        printf("Allocation failure\n");
        exit_status = -1;
        goto END;
    }

    /* Read A from data file */
    scanf(" %39s%*[\n] ", nag_enum_arg);
    /* nag_enum_name_to_value (x04nac).
     * Converts NAG enum member name to value
     */
    uplo = (Nag_UploType) nag_enum_name_to_value(nag_enum_arg);

    if (uplo == Nag_Upper)
    {
        for (i = 1; i <= n; ++i)
        {
            for (j = i; j <= n; ++j)
                scanf(" ( %lf , %lf )", &A(i, j).re, &A(i, j).im);
        }
        scanf("%*[\n] ");
    }
    else
    {
        for (i = 1; i <= n; ++i)
        {
            for (j = 1; j <= i; ++j)

```

```

        scanf(" ( %lf , %lf )", &A(i, j).re, &A(i, j).im);
    }
    scanf("%*[\n] ");
}

/* Compute norm of A */
/* nag_zhe_norm (f16ucc).
 * 1-norm, infinity-norm, Frobenius norm, largest absolute
 * element, complex Hermitian matrix
 */
nag_zhe_norm(order, Nag_OneNorm, uplo, n, a, pda, &anorm, &fail);
if (fail.code != NE_NOERROR)
{
    printf("Error from nag_zhe_norm (f16ucc).\n%s\n", fail.message);
    exit_status = 1;
    goto END;
}
/* Factorize A */

/* nag_zhetrf (f07mrc).
 * Bunch-Kaufman factorization of complex Hermitian
 * indefinite matrix
 */
nag_zhetrf(order, uplo, n, a, pda, ipiv, &fail);
if (fail.code != NE_NOERROR)
{
    printf("Error from nag_zhetrf (f07mrc).\n%s\n", fail.message);
    exit_status = 1;
    goto END;
}
/* Estimate condition number */
/* nag_zhecon (f07muc).
 * Estimate condition number of complex Hermitian indefinite
 * matrix, matrix already factorized by nag_zhetrf (f07mrc)
 */
nag_zhecon(order, uplo, n, a, pda, ipiv, anorm, &rcond,
           &fail);
if (fail.code != NE_NOERROR)
{
    printf("Error from nag_zhecon (f07muc).\n%s\n", fail.message);
    exit_status = 1;
    goto END;
}
/* nag_machine_precision (x02ajc).
 * The machine precision
 */
if (rcond >= nag_machine_precision)
    printf("Estimate of condition number =%11.2e\n", 1.0/rcond);
else
    printf("A is singular to working precision\n");
END:
NAG_FREE(ipiv);
NAG_FREE(a);
return exit_status;
}

```

## 10.2 Program Data

```

nag_zhecon (f07muc) Example Program Data
4                                     :Value of n
Nag_Lower                            :Value of uplo
(-1.36, 0.00)
( 1.58,-0.90) (-8.87, 0.00)
( 2.21, 0.21) (-1.84, 0.03) (-4.63, 0.00)
( 3.91,-1.50) (-1.78,-1.18) ( 0.11,-0.11) (-1.84, 0.00) :End of matrix A

```

### **10.3 Program Results**

nag\_zhecon (f07muc) Example Program Results

Estimate of condition number = 6.68e+00

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