NAG Library Function Document

nag_deviates_studentized_range (g01fmc)

1 Purpose

nag_deviates_studentized_range (g01fmc) returns the deviate associated with the lower tail probability of the distribution of the Studentized range statistic.

2 Specification

```c
#include <nag.h>
#include <nagg01.h>
double nag_deviates_studentized_range (double p, double v, Integer ir,
                                    NagError *fail)
```

3 Description

The externally Studentized range, $q$, for a sample, $x_1, x_2, \ldots, x_r$, is defined as

$$q = \frac{\max(x_i) - \min(x_i)}{\hat{\sigma}_e},$$

where $\hat{\sigma}_e$ is an independent estimate of the standard error of the $x_i$. The most common use of this statistic is in the testing of means from a balanced design. In this case for a set of group means, $\bar{T}_1, \bar{T}_2, \ldots, \bar{T}_r$, the Studentized range statistic is defined to be the difference between the largest and smallest means, $\bar{T}_{\text{largest}}$ and $\bar{T}_{\text{smallest}}$, divided by the square root of the mean-square experimental error, $MS_{\text{error}}$, over the number of observations in each group, $n$, i.e.,

$$q = \frac{\bar{T}_{\text{largest}} - \bar{T}_{\text{smallest}}}{\sqrt{MS_{\text{error}}/n}}.$$ 

The Studentized range statistic can be used as part of a multiple comparisons procedure such as the Newman–Keuls procedure or Duncan’s multiple range test (see Montgomery (1984) and Winer (1970)). For a Studentized range statistic the probability integral, $P(q; v, r)$, for $v$ degrees of freedom and $r$ groups, can be written as:

$$P(q; v, r) = C \int_0^\infty x^{v-1} e^{-x^2/2} \left( r \int_{-\infty}^\infty \phi(y) (\Phi(y) - \Phi(y - qx))^r \right) dx,$$

where

$$C = \frac{v^{v/2}}{\Gamma(v/2)2^{v/2-1}}, \quad \phi(y) = \frac{1}{\sqrt{2\pi}} e^{-y^2/2} \quad \text{and} \quad \Phi(y) = \int_{-\infty}^y \phi(t) dt.$$

For a given probability $p_0$, the deviate $q_0$ is found as the solution to the equation

$$P(q_0; v, r) = p_0,$$

using a root-finding procedure. Initial estimates are found using the approximation given in Lund and Lund (1983) and a simple search procedure.
4 References


5 Arguments

1: \texttt{p} – double \hspace{2cm} \textit{Input}
   
   \textit{On entry:} the lower tail probability for the Studentized range statistic, \( p_0 \).
   
   \textit{Constraint:} \( 0.0 < p < 1.0 \).

2: \texttt{v} – double \hspace{2cm} \textit{Input}
   
   \textit{On entry:} \( v \), the number of degrees of freedom.
   
   \textit{Constraint:} \( v \geq 1.0 \).

3: \texttt{ir} – Integer \hspace{2cm} \textit{Input}
   
   \textit{On entry:} \( r \), the number of groups.
   
   \textit{Constraint:} \( ir \geq 2 \).

4: \texttt{fail} – NagError * \hspace{2cm} \textit{Input/Output}
   
   The NAG error argument (see Section 3.6 in the Essential Introduction).

6 Error Indicators and Warnings

\textbf{NE\_ACCURACY}

Warning – There is some doubt as to whether full accuracy has been achieved.

\textbf{NE\_INIT\_ESTIMATE}

Unable to find initial estimate.

\textbf{NE\_INT}

\textit{On entry,} \( ir = \langle \text{value} \rangle \).

\textit{Constraint:} \( ir \geq 2 \).

\textbf{NE\_INTERNAL\_ERROR}

An internal error has occurred in this function. Check the function call and any array sizes. If the call is correct then please contact NAG for assistance.

\textbf{NE\_REAL}

\textit{On entry,} \( p = \langle \text{value} \rangle \).

\textit{Constraint:} \( 0.0 < p < 1.0 \).

\textit{On entry,} \( v = \langle \text{value} \rangle \).

\textit{Constraint:} \( v \geq 1.0 \).

7 Accuracy

The returned solution, \( q_* \), to equation (1) is determined so that at least one of the following criteria apply.
(a) $|P(q; v, r) - p_0| \leq 0.000005$
(b) $|q_0 - q_i| \leq 0.000005 \times \max(1.0, |q_i|)$.

8 Parallelism and Performance
Not applicable.

9 Further Comments
To obtain the factors for Duncan’s multiple-range test, equation (1) has to be solved for $p_1$, where $p_1 = p_0^{-1}$, so on input $p$ should be set to $p_0^{-1}$.

10 Example
Three values of $p$, $\nu$ and $r$ are read in and the Studentized range deviates or quantiles are computed and printed.

10.1 Program Text
/* nag_deviates_studentized_range (g01fmc) Example Program. *
 * Copyright 2001 Numerical Algorithms Group.
 */
#include <stdio.h>
#include <nag.h>
#include <nag_stdlib.h>
#include <nagg01.h>

int main(void)
{
    /* Scalars */
    double p, v, valq;
    Integer exit_status, i__, ir;
    NagError fail;
    exit_status = 0;
    INIT_FAIL(fail);
    printf("nag_deviates_studentized_range (g01fmc) Example Program Results\n");
    /* Skip heading in data file */
    scanf("%*[\n]");
    printf("%s\n", " p v ir Quantile ");
    for (i__ = 1; i__ <= 3; ++i__)
    {
        scanf("%lf%lf%ld%*[\n]", &p, &v, &ir);
        /* nag_deviates_studentized_range (g01fmc).
         * Computes deviates for the Studentized range statistic
         */
        valq = nag_deviates_studentized_range(p, v, ir, &fail);
        if (fail.code == NE_NOERROR || fail.code == NE_ACCURACY)
        {
            printf("%5.2f%2s%4.1f%1s%1d%1s%10.4f\n", p, "", v, "", ir, "", valq);
        }
        else
        {
            ...
        }
    }

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printf("Error from nag_deviates_studentized_range (g01fmc).\n%s\n",
    fail.message);
exit_status = 1;
goto END;
}

END:
    return exit_status;
}

10.2 Program Data

nag_deviates_studentized_range (g01fmc) Example Program Data
0.95 10.0 5
0.3 60.0 12
0.9 5.0 4

10.3 Program Results

nag_deviates_studentized_range (g01fmc) Example Program Results

<table>
<thead>
<tr>
<th>p</th>
<th>v</th>
<th>ir</th>
<th>Quantile</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.95</td>
<td>10.0</td>
<td>5</td>
<td>4.6543</td>
</tr>
<tr>
<td>0.30</td>
<td>60.0</td>
<td>12</td>
<td>2.8099</td>
</tr>
<tr>
<td>0.90</td>
<td>5.0</td>
<td>4</td>
<td>4.2636</td>
</tr>
</tbody>
</table>