NAG Library Function Document

nag_kelvin_kei (s19adc)

1 Purpose

nag_kelvin_kei (s19adc) returns a value for the Kelvin function $\mathrm{kei}\, x$.

2 Specification

```
#include <nag.h>
#include <nags.h>
double nag_kelvin_kei (double x, NagError *fail)
```

3 Description

nag_kelvin_kei (s19adc) evaluates an approximation to the Kelvin function $\mathrm{kei}\, x$.

Note: for $x < 0$ the function is undefined, so we need only consider $x \geq 0$.

The function is based on several Chebyshev expansions:

For $0 \leq x \leq 1$,

$$\mathrm{kei}\, x = -\frac{\pi}{4} f(t) + \frac{x^2}{4}[-g(t)\log(x) + v(t)]$$

where $f(t)$, $g(t)$ and $v(t)$ are expansions in the variable $t = 2x^4 - 1$;

For $1 < x \leq 3$,

$$\mathrm{kei}\, x = \exp\left(-\frac{9}{8}x\right)u(t)$$

where $u(t)$ is an expansion in the variable $t = x - 2$;

For $x > 3$,

$$\mathrm{kei}\, x = \sqrt{\frac{\pi}{2x}} e^{-x/\sqrt{2}} \left[\left(1 + \frac{1}{x}\right) c(t) \sin\beta + \frac{1}{x} d(t) \cos\beta\right]$$

where $\beta = \frac{x}{\sqrt{2}} + \frac{\pi}{8}$, and $c(t)$ and $d(t)$ are expansions in the variable $t = \frac{6}{x} - 1$.

For $x < 0$, the function is undefined, and hence the function fails and returns zero.

When x is sufficiently close to zero, the result is computed as

$$\mathrm{kei}\, x = -\frac{\pi}{4} + \left(1 - \gamma - \log\left(\frac{x}{2}\right)\right)\frac{x^2}{4}$$

and when x is even closer to zero simply as

$$\mathrm{kei}\, x = -\frac{\pi}{4}.$$

For large x, $\mathrm{kei}\, x$ is asymptotically given by $\sqrt{\frac{\pi}{2x}} e^{-x/\sqrt{2}}$ and this becomes so small that it cannot be computed without underflow and the function fails.

4 References

Abramowitz M and Stegun I A (1972) *Handbook of Mathematical Functions* (3rd Edition) Dover Publications

5 Arguments

1: **x** – double *Input*

On entry: the argument x of the function.

Constraint: $\mathbf{x} \geq 0.0$.

2: **fail** – NagError * *Input/Output*

The NAG error argument (see Section 3.6 in the Essential Introduction).

6 Error Indicators and Warnings

NE_ALLOC_FAIL

Dynamic memory allocation failed.
See Section 3.2.1.2 in the Essential Introduction for further information.

NE_INTERNAL_ERROR

An internal error has occurred in this function. Check the function call and any array sizes. If the call is correct then please contact NAG for assistance.

An unexpected error has been triggered by this function. Please contact NAG.
See Section 3.6.6 in the Essential Introduction for further information.

NE_NO_LICENCE

Your licence key may have expired or may not have been installed correctly.
See Section 3.6.5 in the Essential Introduction for further information.

NE_REAL_ARG_GT

On entry, $\mathbf{x} = \langle value \rangle$. The function returns zero.
Constraint: $\mathbf{x} \leq \langle value \rangle$.
x is too large, the result underflows and the function returns zero.

NE_REAL_ARG_LT

On entry, $\mathbf{x} = \langle value \rangle$.
Constraint: $\mathbf{x} \geq 0.0$.
The function is undefined and returns zero.

7 Accuracy

Let E be the absolute error in the result, and δ be the relative error in the argument. If δ is somewhat larger than the machine representation error, then we have:

$$E \simeq \left| \frac{x}{\sqrt{2}} (-\mathrm{ker}_1\, x + \mathrm{kei}_1\, x) \right| \delta.$$

For small x, errors are attenuated by the function and hence are limited by the ***machine precision***.

For medium and large x, the error behaviour, like the function itself, is oscillatory and hence only absolute accuracy of the function can be maintained. For this range of x, the amplitude of the absolute error decays like $\sqrt{\frac{\pi x}{2}} e^{-x/\sqrt{2}}$, which implies a strong attenuation of error. Eventually, $\mathrm{kei}\, x$, which is asymptotically given by $\sqrt{\frac{\pi}{2x}}e^{-x/\sqrt{2}}$,becomes so small that it cannot be calculated without causing underflow and therefore the function returns zero. Note that for large x, the errors are dominated by those of the standard math library function exp.

8 Parallelism and Performance

Not applicable.

9 Further Comments

Underflow may occur for a few values of x close to the zeros of $\mathrm{kei}\,x$, below the limit which causes a failure with **fail.code** = NE_REAL_ARG_GT.

10 Example

This example reads values of the argument x from a file, evaluates the function at each value of x and prints the results.

10.1 Program Text

```
/* nag_kelvin_kei (s19adc) Example Program.
 *
 * Copyright 2014 Numerical Algorithms Group.
 *
 * Mark 2 revised, 1992.
 */

#include <nag.h>
#include <stdio.h>
#include <nag_stdlib.h>
#include <nags.h>

int main(void)
{
  Integer  exit_status = 0;
  double   x, y;
  NagError fail;

  INIT_FAIL(fail);

  /* Skip heading in data file */
#ifdef _WIN32
  scanf_s("%*[^\n]");
#else
  scanf("%*[^\n]");
#endif
  printf("nag_kelvin_kei (s19adc) Example Program Results\n");
  printf("     x           y\n");
#ifdef _WIN32
  while (scanf_s("%lf", &x) != EOF)
#else
  while (scanf("%lf", &x) != EOF)
#endif
    {
      /* nag_kelvin_kei (s19adc).
       * Kelvin function kei x
       */
      y = nag_kelvin_kei(x, &fail);
      if (fail.code != NE_NOERROR)
        {
          printf("Error from nag_kelvin_kei (s19adc).\n%s\n",
                 fail.message);
          exit_status = 1;
          goto END;
```

```
        }
      printf("%12.3e%12.3e\n", x, y);
    }

 END:
  return exit_status;
}
```

10.2 Program Data

```
nag_kelvin_kei (s19adc) Example Program Data
             0.0
             0.1
             1.0
             2.5
             5.0
             10.0
             15.0
```

10.3 Program Results

```
nag_kelvin_kei (s19adc) Example Program Results
     x           y
   0.000e+00  -7.854e-01
   1.000e-01  -7.769e-01
   1.000e+00  -4.950e-01
   2.500e+00  -1.107e-01
   5.000e+00   1.119e-02
   1.000e+01  -3.075e-04
   1.500e+01   7.963e-06
```
