

NAG Library Function Document

nag_bessel_k0_vector (s18aqc)

1 Purpose

nag_bessel_k0_vector (s18aqc) returns an array of values of the modified Bessel function $K_0(x)$.

2 Specification

```
#include <nag.h>
#include <nags.h>
void nag_bessel_k0_vector (Integer n, const double x[], double f[],
    Integer ivalid[], NagError *fail)
```

3 Description

nag_bessel_k0_vector (s18aqc) evaluates an approximation to the modified Bessel function of the second kind $K_0(x_i)$ for an array of arguments x_i , for $i = 1, 2, \dots, n$.

Note: $K_0(x)$ is undefined for $x \leq 0$ and the function will fail for such arguments.

The function is based on five Chebyshev expansions:

For $0 < x \leq 1$,

$$K_0(x) = -\ln x \sum_{r=0} a_r T_r(t) + \sum_{r=0} b_r T_r(t), \quad \text{where } t = 2x^2 - 1.$$

For $1 < x \leq 2$,

$$K_0(x) = e^{-x} \sum_{r=0} c_r T_r(t), \quad \text{where } t = 2x - 3.$$

For $2 < x \leq 4$,

$$K_0(x) = e^{-x} \sum_{r=0} d_r T_r(t), \quad \text{where } t = x - 3.$$

For $x > 4$,

$$K_0(x) = \frac{e^{-x}}{\sqrt{x}} \sum_{r=0} e_r T_r(t), \quad \text{where } t = \frac{9-x}{1+x}.$$

For x near zero, $K_0(x) \simeq -\gamma - \ln\left(\frac{x}{2}\right)$, where γ denotes Euler's constant. This approximation is used when x is sufficiently small for the result to be correct to *machine precision*.

For large x , where there is a danger of underflow due to the smallness of K_0 , the result is set exactly to zero.

4 References

Abramowitz M and Stegun I A (1972) *Handbook of Mathematical Functions* (3rd Edition) Dover Publications

5 Arguments

- 1: **n** – Integer *Input*
On entry: n , the number of points.
Constraint: $n \geq 0$.
- 2: **x[n]** – const double *Input*
On entry: the argument x_i of the function, for $i = 1, 2, \dots, n$.
Constraint: $x[i - 1] > 0.0$, for $i = 1, 2, \dots, n$.
- 3: **f[n]** – double *Output*
On exit: $K_0(x_i)$, the function values.
- 4: **ivalid[n]** – Integer *Output*
On exit: **ivalid**[$i - 1$] contains the error code for x_i , for $i = 1, 2, \dots, n$.
ivalid[$i - 1$] = 0
 No error.
ivalid[$i - 1$] = 1
 $x_i \leq 0.0$, $K_0(x_i)$ is undefined. **f**[$i - 1$] contains 0.0.
- 5: **fail** – NagError * *Input/Output*
 The NAG error argument (see Section 2.7 in How to Use the NAG Library and its Documentation).

6 Error Indicators and Warnings

NE_ALLOC_FAIL

Dynamic memory allocation failed.

See Section 3.2.1.2 in How to Use the NAG Library and its Documentation for further information.

NE_BAD_PARAM

On entry, argument $\langle value \rangle$ had an illegal value.

NE_INT

On entry, **n** = $\langle value \rangle$.

Constraint: $n \geq 0$.

NE_INTERNAL_ERROR

An internal error has occurred in this function. Check the function call and any array sizes. If the call is correct then please contact NAG for assistance.

An unexpected error has been triggered by this function. Please contact NAG.

See Section 3.6.6 in How to Use the NAG Library and its Documentation for further information.

NE_NO_LICENCE

Your licence key may have expired or may not have been installed correctly.

See Section 3.6.5 in How to Use the NAG Library and its Documentation for further information.

NW_INVALID

On entry, at least one value of **x** was invalid.
Check **ivalid** for more information.

7 Accuracy

Let δ and ϵ be the relative errors in the argument and result respectively.

If δ is somewhat larger than the *machine precision* (i.e., if δ is due to data errors etc.), then ϵ and δ are approximately related by:

$$\epsilon \simeq \left| \frac{xK_1(x)}{K_0(x)} \right| \delta.$$

Figure 1 shows the behaviour of the error amplification factor

$$\left| \frac{xK_1(x)}{K_0(x)} \right|.$$

However, if δ is of the same order as *machine precision*, then rounding errors could make ϵ slightly larger than the above relation predicts.

For small x , the amplification factor is approximately $\left| \frac{1}{\ln x} \right|$, which implies strong attenuation of the error, but in general ϵ can never be less than the *machine precision*.

For large x , $\epsilon \simeq x\delta$ and we have strong amplification of the relative error. Eventually K_0 , which is asymptotically given by $\frac{e^{-x}}{\sqrt{x}}$, becomes so small that it cannot be calculated without underflow and hence the function will return zero. Note that for large x the errors will be dominated by those of the standard function `exp`.

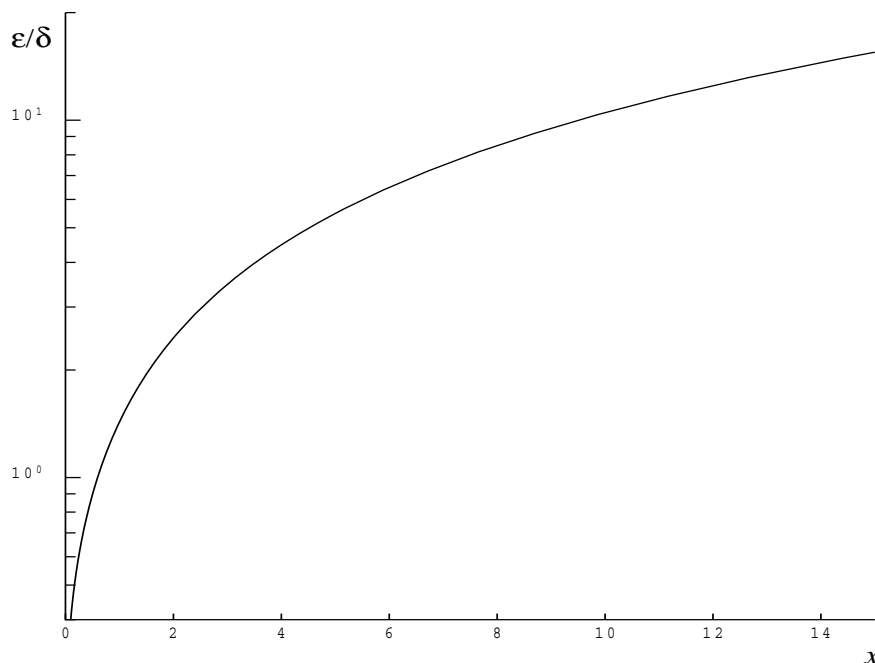


Figure 1

8 Parallelism and Performance

`nag_bessel_k0_vector` (s18aqc) is not threaded in any implementation.

9 Further Comments

None.

10 Example

This example reads values of x from a file, evaluates the function at each value of x_i and prints the results.

10.1 Program Text

```

/* nag_bessel_k0_vector (s18aqc) Example Program.
 *
 * NAGPRODCODE Version.
 *
 * Copyright 2016 Numerical Algorithms Group.
 *
 * Mark 26, 2016.
 */
#include <nag.h>
#include <stdio.h>
#include <nag_stdlib.h>
#include <nags.h>

int main(void)
{
    Integer exit_status = 0;
    Integer i, n;
    double *f = 0, *x = 0;
    Integer *ivalid = 0;
    NagError fail;

    INIT_FAIL(fail);

    /* Skip heading in data file */
#ifdef _WIN32
    scanf_s("%*[\n]");
#else
    scanf("%*[\n]");
#endif

    printf("nag_bessel_k0_vector (s18aqc) Example Program Results\n");
    printf("\n");
    printf("      x          f          ivalid\n");
    printf("\n");
#ifdef _WIN32
    scanf_s("%" NAG_IFMT " ", &n);
#else
    scanf("%" NAG_IFMT " ", &n);
#endif
#ifdef _WIN32
    scanf_s("%*[\n]");
#else
    scanf("%*[\n]");
#endif

    /* Allocate memory */
    if (!(x = NAG_ALLOC(n, double)) ||
        !(f = NAG_ALLOC(n, double)) || !(ivalid = NAG_ALLOC(n, Integer)))
    {
        printf("Allocation failure\n");
        exit_status = -1;
        goto END;
    }

    for (i = 0; i < n; i++)
#ifdef _WIN32
        scanf_s("%lf", &x[i]);

```

```

#else
    scanf("%lf", &x[i]);
#endif
#ifdef _WIN32
    scanf_s("%*[^\\n]");
#else
    scanf("%*[^\\n]");
#endif

/* nag_bessel_k0_vector (s18aqc).
 * modified Bessel Function K0(x)
 */
nag_bessel_k0_vector(n, x, f, ivalid, &fail);
if (fail.code != NE_NOERROR && fail.code != NW_INVALID) {
    printf("Error from nag_bessel_k0_vector (s18aqc).\n%s\n", fail.message);
    exit_status = 1;
    goto END;
}

for (i = 0; i < n; i++)
    printf(" %11.3e %11.3e %4" NAG_IFMT "\n", x[i], f[i], ivalid[i]);

END:
    NAG_FREE(f);
    NAG_FREE(x);
    NAG_FREE(ivalid);

    return exit_status;
}

```

10.2 Program Data

nag_bessel_k0_vector (s18aqc) Example Program Data

10

0.4 0.6 1.4 1.6 2.5 3.5 6.0 8.0 10.0 1000.0

10.3 Program Results

nag_bessel_k0_vector (s18aqc) Example Program Results

x	f	ivalid
4.000e-01	1.115e+00	0
6.000e-01	7.775e-01	0
1.400e+00	2.437e-01	0
1.600e+00	1.880e-01	0
2.500e+00	6.235e-02	0
3.500e+00	1.960e-02	0
6.000e+00	1.244e-03	0
8.000e+00	1.465e-04	0
1.000e+01	1.778e-05	0
1.000e+03	0.000e+00	0
