

NAG Library Routine Document

F08GEF (DSPTRD)

Note: before using this routine, please read the Users' Note for your implementation to check the interpretation of *bold italicised* terms and other implementation-dependent details.

1 Purpose

F08GEF (DSPTRD) reduces a real symmetric matrix to tridiagonal form, using packed storage.

2 Specification

```
SUBROUTINE F08GEF (UPLO, N, AP, D, E, TAU, INFO)
  INTEGER          N, INFO
  REAL (KIND=nag_wp) AP(*), D(N), E(N-1), TAU(N-1)
  CHARACTER(1)    UPLO
```

The routine may be called by its LAPACK name *dsptrd*.

3 Description

F08GEF (DSPTRD) reduces a real symmetric matrix A , held in packed storage, to symmetric tridiagonal form T by an orthogonal similarity transformation: $A = QTQ^T$.

The matrix Q is not formed explicitly but is represented as a product of $n - 1$ elementary reflectors (see the F08 Chapter Introduction for details). Routines are provided to work with Q in this representation (see Section 8).

4 References

Golub G H and Van Loan C F (1996) *Matrix Computations* (3rd Edition) Johns Hopkins University Press, Baltimore

5 Parameters

- 1: UPLO – CHARACTER(1) *Input*
On entry: indicates whether the upper or lower triangular part of A is stored.
 UPLO = 'U'
 The upper triangular part of A is stored.
 UPLO = 'L'
 The lower triangular part of A is stored.
Constraint: UPLO = 'U' or 'L'.
- 2: N – INTEGER *Input*
On entry: n , the order of the matrix A .
Constraint: $N \geq 0$.
- 3: AP(*) – REAL (KIND=nag_wp) array *Input/Output*
Note: the dimension of the array AP must be at least $\max(1, N \times (N + 1)/2)$.
On entry: the upper or lower triangle of the n by n symmetric matrix A , packed by columns.

More precisely,

if UPLO = 'U', the upper triangle of A must be stored with element A_{ij} in $AP(i + j(j - 1)/2)$ for $i \leq j$;

if UPLO = 'L', the lower triangle of A must be stored with element A_{ij} in $AP(i + (2n - j)(j - 1)/2)$ for $i \geq j$.

On exit: AP is overwritten by the tridiagonal matrix T and details of the orthogonal matrix Q .

4: D(N) – REAL (KIND=nag_wp) array Output

On exit: the diagonal elements of the tridiagonal matrix T .

5: E(N – 1) – REAL (KIND=nag_wp) array Output

On exit: the off-diagonal elements of the tridiagonal matrix T .

6: TAU(N – 1) – REAL (KIND=nag_wp) array Output

On exit: further details of the orthogonal matrix Q .

7: INFO – INTEGER Output

On exit: INFO = 0 unless the routine detects an error (see Section 6).

6 Error Indicators and Warnings

Errors or warnings detected by the routine:

INFO < 0

If INFO = $-i$, argument i had an illegal value. An explanatory message is output, and execution of the program is terminated.

7 Accuracy

The computed tridiagonal matrix T is exactly similar to a nearby matrix $(A + E)$, where

$$\|E\|_2 \leq c(n)\epsilon\|A\|_2,$$

$c(n)$ is a modestly increasing function of n , and ϵ is the *machine precision*.

The elements of T themselves may be sensitive to small perturbations in A or to rounding errors in the computation, but this does not affect the stability of the eigenvalues and eigenvectors.

8 Further Comments

The total number of floating point operations is approximately $\frac{4}{3}n^3$.

To form the orthogonal matrix Q F08GEF (DSPTRD) may be followed by a call to F08GFF (DOPGTR):

```
CALL DOPGTR(UPLO,N,AP,TAU,Q,LDQ,WORK,INFO)
```

To apply Q to an n by p real matrix C F08GEF (DSPTRD) may be followed by a call to F08GGF (DOPMTR). For example,

```
CALL DOPMTR('Left',UPLO,'No Transpose',N,P,AP,TAU,C,LDC,WORK, &
           INFO)
```

forms the matrix product QC .

The complex analogue of this routine is F08GSF (ZHPTRD).

9 Example

This example reduces the matrix A to tridiagonal form, where

$$A = \begin{pmatrix} 2.07 & 3.87 & 4.20 & -1.15 \\ 3.87 & -0.21 & 1.87 & 0.63 \\ 4.20 & 1.87 & 1.15 & 2.06 \\ -1.15 & 0.63 & 2.06 & -1.81 \end{pmatrix},$$

using packed storage.

9.1 Program Text

Program f08gefe

```

!      F08GEF Example Program Text
!
!      Mark 24 Release. NAG Copyright 2012.
!
!      .. Use Statements ..
!      Use nag_library, Only: dsptd, nag_wp
!      .. Implicit None Statement ..
!      Implicit None
!      .. Parameters ..
!      Integer, Parameter          :: nin = 5, nout = 6
!      .. Local Scalars ..
!      Integer                    :: i, info, j, n
!      Character (1)              :: uplo
!      .. Local Arrays ..
!      Real (Kind=nag_wp), Allocatable :: ap(:), d(:), e(:), tau(:)
!      .. Intrinsic Procedures ..
!      Intrinsic                  :: abs
!      .. Executable Statements ..
!      Write (nout,*) 'F08GEF Example Program Results'
!      Skip heading in data file
!      Read (nin,*)
!      Read (nin,*) n

!      Allocate (ap(n*(n+1)/2),d(n),e(n-1),tau(n-1))

!      Read A from data file and copy A into AW

!      Read (nin,*) uplo
!      If (uplo=='U') Then
!         Read (nin,*)((ap(i+j*(j-1)/2),j=i,n),i=1,n)
!      Else If (uplo=='L') Then
!         Read (nin,*)((ap(i+(2*n-j)*(j-1)/2),j=1,i),i=1,n)
!      End If

!      Reduce A to tridiagonal form
!      The NAG name equivalent of dsptd is f08gef
!      Call dsptd(uplo,n,ap,d,e,tau,info)

!      If (info==0) Then
!         Print the diagonal and off-diagonal of tridiagonal T.
!         The absolute value of E is printed since this can vary by a change of
!         sign (correspondng to multiplying through a column of Q by -1).

!         Write (nout,*)
!         Write (nout,*) &
!           'Diagonal and off-diagonal elements of tridiagonal form'
!         Write (nout,*)
!         Write (nout,99999) 'i', 'D', 'E'
!         Do i = 1, n - 1
!            Write (nout,99998) i, d(i), abs(e(i))
!         End Do
!         Write (nout,99998) n, d(n)

!      Else

```

```

        Write (nout,99997) info
      End If

99999 Format (5X,A,9X,A,12X,A)
99998 Format (1X,I5,2(1X,F12.5))
99997 Format (1X,'** DSPTRD/F08GEF retuned with INFO = ',I10)

      End Program f08gefe

```

9.2 Program Data

```

F08GEF Example Program Data
4                               :Value of N
'L'                             :Value of UPLO
2.07
3.87 -0.21
4.20  1.87  1.15
-1.15  0.63  2.06 -1.81      :End of matrix A

```

9.3 Program Results

```

F08GEF Example Program Results

Diagonal and off-diagonal elements of tridiagonal form

```

i	D	E
1	2.07000	5.82575
2	1.47409	2.62405
3	-0.64916	0.91627
4	-1.69493	
