

NAG Library Routine Document

F07USF (ZTPTRS)

Note: before using this routine, please read the Users' Note for your implementation to check the interpretation of *bold italicised* terms and other implementation-dependent details.

1 Purpose

F07USF (ZTPTRS) solves a complex triangular system of linear equations with multiple right-hand sides, $AX = B$, $A^T X = B$ or $A^H X = B$, using packed storage.

2 Specification

SUBROUTINE F07USF (UPLO, TRANS, DIAG, N, NRHS, AP, B, LDB, INFO)

INTEGER N, NRHS, LDB, INFO
 COMPLEX (KIND=nag_wp) AP(*), B(LDB,*)
 CHARACTER(1) UPLO, TRANS, DIAG

The routine may be called by its LAPACK name *ztptrs*.

3 Description

F07USF (ZTPTRS) solves a complex triangular system of linear equations $AX = B$, $A^T X = B$ or $A^H X = B$, using packed storage.

4 References

Golub G H and Van Loan C F (1996) *Matrix Computations* (3rd Edition) Johns Hopkins University Press, Baltimore

Higham N J (1989) The accuracy of solutions to triangular systems *SIAM J. Numer. Anal.* **26** 1252–1265

5 Parameters

- 1: UPLO – CHARACTER(1) *Input*
On entry: specifies whether A is upper or lower triangular.
 UPLO = 'U'
 A is upper triangular.
 UPLO = 'L'
 A is lower triangular.
Constraint: UPLO = 'U' or 'L'.
- 2: TRANS – CHARACTER(1) *Input*
On entry: indicates the form of the equations.
 TRANS = 'N'
 The equations are of the form $AX = B$.
 TRANS = 'T'
 The equations are of the form $A^T X = B$.

TRANS = 'C'

The equations are of the form $A^H X = B$.

Constraint: TRANS = 'N', 'T' or 'C'.

- 3: DIAG – CHARACTER(1) *Input*
On entry: indicates whether A is a nonunit or unit triangular matrix.
 DIAG = 'N'
 A is a nonunit triangular matrix.
 DIAG = 'U'
 A is a unit triangular matrix; the diagonal elements are not referenced and are assumed to be 1.
Constraint: DIAG = 'N' or 'U'.
- 4: N – INTEGER *Input*
On entry: n , the order of the matrix A .
Constraint: $N \geq 0$.
- 5: NRHS – INTEGER *Input*
On entry: r , the number of right-hand sides.
Constraint: NRHS ≥ 0 .
- 6: AP(*) – COMPLEX (KIND=nag_wp) array *Input*
Note: the dimension of the array AP must be at least $\max(1, N \times (N + 1)/2)$.
On entry: the n by n triangular matrix A , packed by columns.
 More precisely,
 if UPLO = 'U', the upper triangle of A must be stored with element A_{ij} in
 AP($i + j(j - 1)/2$) for $i \leq j$;
 if UPLO = 'L', the lower triangle of A must be stored with element A_{ij} in
 AP($i + (2n - j)(j - 1)/2$) for $i \geq j$.
 If DIAG = 'U', the diagonal elements of A are assumed to be 1, and are not referenced; the same storage scheme is used whether DIAG = 'N' or 'U'.
- 7: B(LDB,*) – COMPLEX (KIND=nag_wp) array *Input/Output*
Note: the second dimension of the array B must be at least $\max(1, \text{NRHS})$.
On entry: the n by r right-hand side matrix B .
On exit: the n by r solution matrix X .
- 8: LDB – INTEGER *Input*
On entry: the first dimension of the array B as declared in the (sub)program from which F07USF (ZTPTRS) is called.
Constraint: LDB $\geq \max(1, N)$.
- 9: INFO – INTEGER *Output*
On exit: INFO = 0 unless the routine detects an error (see Section 6).

6 Error Indicators and Warnings

Errors or warnings detected by the routine:

INFO < 0

If INFO = $-i$, the i th parameter had an illegal value. An explanatory message is output, and execution of the program is terminated.

INFO > 0

If INFO = i , $a(i, i)$ is exactly zero; A is singular and the solution has not been computed.

7 Accuracy

The solutions of triangular systems of equations are usually computed to high accuracy. See Higham (1989).

For each right-hand side vector b , the computed solution x is the exact solution of a perturbed system of equations $(A + E)x = b$, where

$$|E| \leq c(n)\epsilon|A|,$$

$c(n)$ is a modest linear function of n , and ϵ is the *machine precision*.

If \hat{x} is the true solution, then the computed solution x satisfies a forward error bound of the form

$$\frac{\|x - \hat{x}\|_\infty}{\|x\|_\infty} \leq c(n) \operatorname{cond}(A, x)\epsilon, \quad \text{provided} \quad c(n) \operatorname{cond}(A, x)\epsilon < 1,$$

where $\operatorname{cond}(A, x) = \frac{\|A^{-1}\|_\infty \|A\|_\infty \|x\|_\infty}{\|x\|_\infty}$.

Note that $\operatorname{cond}(A, x) \leq \operatorname{cond}(A) = \|A^{-1}\|_\infty \|A\|_\infty \leq \kappa_\infty(A)$; $\operatorname{cond}(A, x)$ can be much smaller than $\operatorname{cond}(A)$ and it is also possible for $\operatorname{cond}(A^H)$, which is the same as $\operatorname{cond}(A^T)$, to be much larger (or smaller) than $\operatorname{cond}(A)$.

Forward and backward error bounds can be computed by calling F07UVF (ZTPRFS), and an estimate for $\kappa_\infty(A)$ can be obtained by calling F07UUF (ZTPCON) with NORM = 'I'.

8 Further Comments

The total number of real floating point operations is approximately $4n^2r$.

The real analogue of this routine is F07UEF (DTPTRS).

9 Example

This example solves the system of equations $AX = B$, where

$$A = \begin{pmatrix} 4.78 + 4.56i & 0.00 + 0.00i & 0.00 + 0.00i & 0.00 + 0.00i \\ 2.00 - 0.30i & -4.11 + 1.25i & 0.00 + 0.00i & 0.00 + 0.00i \\ 2.89 - 1.34i & 2.36 - 4.25i & 4.15 + 0.80i & 0.00 + 0.00i \\ -1.89 + 1.15i & 0.04 - 3.69i & -0.02 + 0.46i & 0.33 - 0.26i \end{pmatrix}$$

and

$$B = \begin{pmatrix} -14.78 - 32.36i & -18.02 + 28.46i \\ 2.98 - 2.14i & 14.22 + 15.42i \\ -20.96 + 17.06i & 5.62 + 35.89i \\ 9.54 + 9.91i & -16.46 - 1.73i \end{pmatrix},$$

using packed storage for A .

9.1 Program Text

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Program f07usfe

!      F07USF Example Program Text

!      Mark 24 Release. NAG Copyright 2012.

!      .. Use Statements ..
Use nag_library, Only: nag_wp, x04dbf, ztptrs
!      .. Implicit None Statement ..
Implicit None
!      .. Parameters ..
Integer, Parameter          :: nin = 5, nout = 6
Character (1), Parameter   :: diag = 'N', trans = 'N'
!      .. Local Scalars ..
Integer                    :: i, ifail, info, j, ldb, n, nrhs
Character (1)              :: uplo
!      .. Local Arrays ..
Complex (Kind=nag_wp), Allocatable :: ap(:), b(:, :)
Character (1)              :: clabs(1), rlabs(1)
!      .. Executable Statements ..
Write (nout,*) 'F07USF Example Program Results'
Skip heading in data file
Read (nin,*)
Read (nin,*) n, nrhs
ldb = n
Allocate (ap(n*(n+1)/2),b(ldb,nrhs))

!      Read A and B from data file

Read (nin,*) uplo
If (uplo=='U') Then
  Read (nin,*)((ap(i+j*(j-1)/2),j=i,n),i=1,n)
Else If (uplo=='L') Then
  Read (nin,*)((ap(i+(2*n-j)*(j-1)/2),j=1,i),i=1,n)
End If
Read (nin,*)(b(i,1:nrhs),i=1,n)

!      Compute solution
!      The NAG name equivalent of ztptrs is f07usf
Call ztptrs(uplo,trans,diag,n,nrhs,ap,b,ldb,info)

!      Print solution

Write (nout,*)
Flush (nout)
If (info==0) Then

!      ifail: behaviour on error exit
!      =0 for hard exit, =1 for quiet-soft, =-1 for noisy-soft
ifail = 0
Call x04dbf('General',' ',n,nrhs,b,ldb,'Bracketed','F7.4', &
  'Solution(s)','Integer',rlabs,'Integer',clabs,80,0,ifail)

Else
  Write (nout,*) 'A is singular'
End If

End Program f07usfe

```

9.2 Program Data

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F07USF Example Program Data
  4  2                                     :Values of N and NRHS
  'L'                                     :Value of UPLO
( 4.78, 4.56)
( 2.00,-0.30) (-4.11, 1.25)
( 2.89,-1.34) ( 2.36,-4.25) ( 4.15, 0.80)

```

```
(-1.89, 1.15) ( 0.04,-3.69) (-0.02, 0.46) ( 0.33,-0.26) :End of matrix A
(-14.78,-32.36) (-18.02, 28.46)
( 2.98, -2.14) ( 14.22, 15.42)
(-20.96, 17.06) ( 5.62, 35.89)
( 9.54, 9.91) (-16.46, -1.73) :End of matrix B
```

9.3 Program Results

F07USF Example Program Results

Solution(s)

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           1           2
1 (-5.0000,-2.0000) ( 1.0000, 5.0000)
2 (-3.0000,-1.0000) (-2.0000,-2.0000)
3 ( 2.0000, 1.0000) ( 3.0000, 4.0000)
4 ( 4.0000, 3.0000) ( 4.0000,-3.0000)
```
