NAG Library Routine Document

D02UCF

Note: before using this routine, please read the Users' Note for your implementation to check the interpretation of **bold italicised** terms and other implementation-dependent details.

1   Purpose

D02UCF returns the Chebyshev Gauss–Lobatto grid points on $[a, b]$.

2   Specification

```
SUBROUTINE D02UCF (N, A, B, X, IFAIL)
INTEGER          N, IFAIL
REAL (KIND=nag_wp) A, B, X(N+1)
```

3   Description

D02UCF returns the Chebyshev Gauss–Lobatto grid points on $[a, b]$. The Chebyshev Gauss–Lobatto points on $[-1, 1]$ are computed as $t_i = -\cos\left(\frac{(i-1)\pi}{n}\right)$, for $i = 1, 2, \ldots, n+1$. The Chebyshev Gauss–Lobatto points on an arbitrary domain $[a, b]$ are:

$$x_i = \frac{b-a}{2}t_i + \frac{a+b}{2}, \quad i = 1, 2, \ldots, n+1.$$

4   References

Trefethen L N (2000) *Spectral Methods in MATLAB* SIAM

5   Parameters

1:    N – INTEGER   *Input*

On entry: n, where the number of grid points is $n + 1$. This is also the largest order of Chebyshev polynomial in the Chebyshev series to be computed.

Constraint: N > 0 and N is even.

2:    A – REAL (KIND=nag_wp)   *Input*

On entry: a, the lower bound of domain $[a, b]$.

Constraint: A < B.

3:    B – REAL (KIND=nag_wp)   *Input*

On entry: b, the upper bound of domain $[a, b]$.

Constraint: B > A.

4:    X(N + 1) – REAL (KIND=nag_wp) array   *Output*

On exit: the Chebyshev Gauss–Lobatto grid points, x_i, for $i = 1, 2, \ldots, n+1$, on $[a, b]$.

5:    IFAIL – INTEGER   *Input/Output*

On entry: IFAIL must be set to 0, -1 or 1. If you are unfamiliar with this parameter you should refer to Section 3.3 in the Essential Introduction for details.

For environments where it might be inappropriate to halt program execution when an error is detected, the value -1 or 1 is recommended. If the output of error messages is undesirable, then the value 1 is recommended. Otherwise, if you are not familiar with this parameter, the recommended value is 0. **When the value -1 or 1 is used it is essential to test the value of IFAIL on exit.**

On exit: IFAIL $= 0$ unless the routine detects an error or a warning has been flagged (see Section 6).

6   Error Indicators and Warnings

If on entry IFAIL $= 0$ or -1, explanatory error messages are output on the current error message unit (as defined by X04AAF).

Errors or warnings detected by the routine:

IFAIL $= 1$

> On entry, N $= \langle value \rangle$.
> Constraint: N > 0.

> On entry, N $= \langle value \rangle$.
> Constraint: N is even.

IFAIL $= 2$

> On entry, A $= \langle value \rangle$ and B $= \langle value \rangle$.
> Constraint: A $<$ B.

IFAIL $= -99$

> An unexpected error has been triggered by this routine. Please contact NAG.

> See Section 3.8 in the Essential Introduction for further information.

IFAIL $= -399$

> Your licence key may have expired or may not have been installed correctly.

> See Section 3.7 in the Essential Introduction for further information.

IFAIL $= -999$

> Dynamic memory allocation failed.

> See Section 3.6 in the Essential Introduction for further information.

7   Accuracy

The Chebyshev Gauss$-$Lobatto grid points computed should be accurate to within a small multiple of *machine precision*.

8   Parallelism and Performance

Not applicable.

9   Further Comments

The number of operations is of the order $n \log (n)$ and there are no internal memory requirements; thus the computation remains efficient and practical for very fine discretizations (very large values of n).

10 Example

See Section 10 in D02UEF.