NAG Library Routine Document

F08WSF (ZGGHRD)

Note: before using this routine, please read the Users' Note for your implementation to check the interpretation of ***bold italicised*** terms and other implementation-dependent details.

1 Purpose

F08WSF (ZGGHRD) reduces a pair of complex matrices (A, B), where B is upper triangular, to the generalized upper Hessenberg form using unitary transformations.

2 Specification

```
SUBROUTINE F08WSF (COMPQ, COMPZ, N, ILO, IHI, A, LDA, B, LDB, Q, LDQ, Z,    &
                   LDZ, INFO)

INTEGER                N, ILO, IHI, LDA, LDB, LDQ, LDZ, INFO
COMPLEX (KIND=nag_wp) A(LDA,*), B(LDB,*), Q(LDQ,*), Z(LDZ,*)
CHARACTER(1)           COMPQ, COMPZ
```

The routine may be called by its LAPACK name ***zgghrd***.

3 Description

F08WSF (ZGGHRD) is usually the third step in the solution of the complex generalized eigenvalue problem

$$Ax = \lambda Bx.$$

The (optional) first step balances the two matrices using F08WVF (ZGGBAL). In the second step, matrix B is reduced to upper triangular form using the QR factorization routine F08ASF (ZGEQRF) and this unitary transformation Q is applied to matrix A by calling F08AUF (ZUNMQR).

F08WSF (ZGGHRD) reduces a pair of complex matrices (A, B), where B is triangular, to the generalized upper Hessenberg form using unitary transformations. This two-sided transformation is of the form

$$\begin{aligned} Q^{\mathrm{H}}AZ &= H \\ Q^{\mathrm{H}}BZ &= T \end{aligned}$$

where H is an upper Hessenberg matrix, T is an upper triangular matrix and Q and Z are unitary matrices determined as products of Givens rotations. They may either be formed explicitly, or they may be postmultiplied into input matrices Q_1 and Z_1, so that

$$\begin{aligned} Q_1AZ_1^{\mathrm{H}} &= (Q_1Q)H(Z_1Z)^{\mathrm{H}}, \\ Q_1BZ_1^{\mathrm{H}} &= (Q_1Q)T(Z_1Z)^{\mathrm{H}}. \end{aligned}$$

4 References

Golub G H and Van Loan C F (1996) *Matrix Computations* (3rd Edition) Johns Hopkins University Press, Baltimore

Moler C B and Stewart G W (1973) An algorithm for generalized matrix eigenproblems *SIAM J. Numer. Anal.* **10** 241–256

5 Parameters

1: COMPQ – CHARACTER(1) *Input*

On entry: specifies the form of the computed unitary matrix Q.

COMPQ = 'N'
Do not compute Q.

COMPQ = 'I'
The unitary matrix Q is returned.

COMPQ = 'V'
Q must contain a unitary matrix Q_1, and the product $Q_1 Q$ is returned.

Constraint: COMPQ = 'N', 'I' or 'V'.

2: COMPZ – CHARACTER(1) *Input*

On entry: specifies the form of the computed unitary matrix Z.

COMPZ = 'N'
Do not compute Z.

COMPZ = 'V'
Z must contain a unitary matrix Z_1, and the product $Z_1 Z$ is returned.

COMPZ = 'I'
The unitary matrix Z is returned.

Constraint: COMPZ = 'N', 'V' or 'I'.

3: N – INTEGER *Input*

On entry: n, the order of the matrices A and B.

Constraint: $\mathrm{N} \geq 0$.

4: ILO – INTEGER *Input*
5: IHI – INTEGER *Input*

On entry: i_{lo} and i_{hi} as determined by a previous call to F08WVF (ZGGBAL). Otherwise, they should be set to 1 and n, respectively.

Constraints:

if $\mathrm{N} > 0$, $1 \leq \mathrm{ILO} \leq \mathrm{IHI} \leq \mathrm{N}$;
if $\mathrm{N} = 0$, $\mathrm{ILO} = 1$ and $\mathrm{IHI} = 0$.

6: $\mathrm{A}(\mathrm{LDA}, *)$ – COMPLEX (KIND=nag_wp) array *Input/Output*

Note: the second dimension of the array A must be at least $\max(1, \mathrm{N})$.

On entry: the matrix A of the matrix pair (A, B). Usually, this is the matrix A returned by F08AUF (ZUNMQR).

On exit: A is overwritten by the upper Hessenberg matrix H.

7: LDA – INTEGER *Input*

On entry: the first dimension of the array A as declared in the (sub)program from which F08WSF (ZGGHRD) is called.

Constraint: $\mathrm{LDA} \geq \max(1, \mathrm{N})$.

8: B(LDB, $*$) – COMPLEX (KIND=nag_wp) array *Input/Output*

Note: the second dimension of the array B must be at least max(1, N).

On entry: the upper triangular matrix B of the matrix pair (A, B). Usually, this is the matrix B returned by the QR factorization routine F08ASF (ZGEQRF).

On exit: B is overwritten by the upper triangular matrix T.

9: LDB – INTEGER *Input*

On entry: the first dimension of the array B as declared in the (sub)program from which F08WSF (ZGGHRD) is called.

Constraint: LDB $\geq$ max(1, N).

10: Q(LDQ, $*$) – COMPLEX (KIND=nag_wp) array *Input/Output*

Note: the second dimension of the array Q must be at least max(1, N) if COMPQ = 'I' or 'V' and at least 1 if COMPQ = 'N'.

On entry: if COMPQ = 'V', Q must contain a unitary matrix Q_1.

If COMPQ = 'N', Q is not referenced.

On exit: if COMPQ = 'I', Q contains the unitary matrix Q.

Iif COMPQ = 'V', Q is overwritten by Q_1Q.

11: LDQ – INTEGER *Input*

On entry: the first dimension of the array Q as declared in the (sub)program from which F08WSF (ZGGHRD) is called.

Constraints:

if COMPQ = 'I' or 'V', LDQ $\geq$ max(1, N);
if COMPQ = 'N', LDQ ≥ 1.

12: Z(LDZ, $*$) – COMPLEX (KIND=nag_wp) array *Input/Output*

Note: the second dimension of the array Z must be at least max(1, N) if COMPZ = 'V' or 'I' and at least 1 if COMPZ = 'N'.

On entry: if COMPZ = 'V', Z must contain a unitary matrix Z_1.

If COMPZ = 'N', Z is not referenced.

On exit: if COMPZ = 'I', Z contains the unitary matrix Z.

If COMPZ = 'V', Z is overwritten by Z_1Z.

13: LDZ – INTEGER *Input*

On entry: the first dimension of the array Z as declared in the (sub)program from which F08WSF (ZGGHRD) is called.

Constraints:

if COMPZ = 'V' or 'I', LDZ $\geq$ max(1, N);
if COMPZ = 'N', LDZ ≥ 1.

14: INFO – INTEGER *Output*

On exit: INFO = 0 unless the routine detects an error (see Section 6).

6 Error Indicators and Warnings

INFO < 0

If INFO $= -i$, argument i had an illegal value. An explanatory message is output, and execution of the program is terminated.

7 Accuracy

The reduction to the generalized Hessenberg form is implemented using unitary transformations which are backward stable.

8 Parallelism and Performance

Not applicable.

9 Further Comments

This routine is usually followed by F08XSF (ZHGEQZ) which implements the QZ algorithm for computing generalized eigenvalues of a reduced pair of matrices.

The real analogue of this routine is F08WEF (DGGHRD).

10 Example

See Section 10 in F08XSF (ZHGEQZ) and F08YXF (ZTGEVC).