# **NAG Library Routine Document**

# G05PEF

Note: before using this routine, please read the Users' Note for your implementation to check the interpretation of *bold italicised* terms and other implementation-dependent details.

# 1 Purpose

G05PEF generates a given number of terms of a type II AGARCH(p,q) process (see Engle and Ng (1993)).

# 2 Specification

```
SUBROUTINE G05PEF (DIST, NUM, IP, IQ, THETA, GAMMA, DF, HT, ET, FCALL,
R, LR, STATE, IFAIL)
INTEGER NUM, IP, IQ, DF, LR, STATE(*), IFAIL
REAL (KIND=nag_wp) THETA(IQ+IP+1), GAMMA, HT(NUM), ET(NUM), R(LR)
LOGICAL FCALL
CHARACTER(1) DIST
```

# **3** Description

A type II AGARCH(p,q) process can be represented by:

$$h_t = \alpha_0 + \sum_{i=1}^q \alpha_i (|\epsilon_{t-i}| + \gamma \epsilon_{t-i})^2 + \sum_{i=1}^p \beta_i h_{t-i}, \quad t = 1, 2, \dots, T;$$

where  $\epsilon_t \mid \psi_{t-1} = N(0, h_t)$  or  $\epsilon_t \mid \psi_{t-1} = S_t(df, h_t)$ . Here  $S_t$  is a standardized Student's *t*-distribution with df degrees of freedom and variance  $h_t$ , T is the number of observations in the sequence,  $\epsilon_t$  is the observed value of the GARCH(p, q) process at time t,  $h_t$  is the conditional variance at time t, and  $\psi_t$  the set of all information up to time t. Symmetric GARCH sequences are generated when  $\gamma$  is zero, otherwise asymmetric GARCH sequences are generated with  $\gamma$  specifying the amount by which positive (or negative) shocks are to be enhanced.

One of the initialization routines G05KFF (for a repeatable sequence if computed sequentially) or G05KGF (for a non-repeatable sequence) must be called prior to the first call to G05PEF.

# 4 References

Bollerslev T (1986) Generalised autoregressive conditional heteroskedasticity *Journal of Econometrics* **31** 307–327

Engle R (1982) Autoregressive conditional heteroskedasticity with estimates of the variance of United Kingdom inflation *Econometrica* **50** 987–1008

Engle R and Ng V (1993) Measuring and testing the impact of news on volatility *Journal of Finance* 48 1749–1777

Hamilton J (1994) Time Series Analysis Princeton University Press

# **5** Parameters

1: DIST - CHARACTER(1)

On entry: the type of distribution to use for  $\epsilon_t$ .

DIST = 'N'

A Normal distribution is used.

Input

Input

Input

Input

Input

DIST = 'T'A Student's *t*-distribution is used. Constraint: DIST = 'N' or 'T'.

2: NUM - INTEGER On entry: T, the number of terms in the sequence.

*Constraint*: NUM  $\geq$  0.

IP - INTEGER 3:

> On entry: the number of coefficients,  $\beta_i$ , for i = 1, 2, ..., p. *Constraint*:  $IP \ge 0$ .

IQ - INTEGER 4:

> On entry: the number of coefficients,  $\alpha_i$ , for i = 1, 2, ..., q. *Constraint*:  $IQ \ge 1$ .

THETA(IQ + IP + 1) – REAL (KIND=nag wp) array 5:

On entry: the first element must contain the coefficient  $\alpha_o$ , the next IQ elements must contain the coefficients  $\alpha_i$ , for i = 1, 2, ..., q. The remaining IP elements must contain the coefficients  $\beta_i$ , for  $j=1,2,\ldots,p.$ 

Constraints:

$$\sum_{i=2}^{IQ+IP+1} \text{THETA}(i) < 1.0;$$
  
THETA(i)  $\ge 0.0$ , for  $i = 2, 3, ..., \text{IP} + \text{IQ} + 1$ .

6: GAMMA – REAL (KIND=nag wp)

On entry: the asymmetry parameter  $\gamma$  for the GARCH(p,q) sequence.

7: DF - INTEGER

On entry: the number of degrees of freedom for the Student's t-distribution.

If DIST = 'N', DF is not referenced.

Constraint: if DIST = 'T', DF > 2.

HT(NUM) - REAL (KIND=nag\_wp) array 8: Output On exit: the conditional variances  $h_t$ , for t = 1, 2, ..., T, for the GARCH(p, q) sequence.

ET(NUM) - REAL (KIND=nag wp) array 9: On exit: the observations  $\epsilon_t$ , for t = 1, 2, ..., T, for the GARCH(p, q) sequence.

- FCALL LOGICAL 10: On entry: if FCALL = .TRUE., a new sequence is to be generated, otherwise a given sequence is to be continued using the information in R.
- 11: R(LR) - REAL (KIND=nag wp) array Input/Output On entry: the array contains information required to continue a sequence if FCALL = .FALSE.. On exit: contains information that can be used in a subsequent call of G05PEF, with FCALL = .FALSE..

Input

Input

Input

Output

12: LR – INTEGER

On entry: the dimension of the array R as declared in the (sub)program from which G05PEF is called.

*Constraint*:  $LR \ge 2 \times (IP + IQ + 2)$ .

13: STATE(\*) – INTEGER array

**Note**: the actual argument supplied **must** be the array STATE supplied to the initialization routines G05KFF or G05KGF.

On entry: contains information on the selected base generator and its current state.

On exit: contains updated information on the state of the generator.

```
14: IFAIL – INTEGER
```

On entry: IFAIL must be set to 0, -1 or 1. If you are unfamiliar with this parameter you should refer to Section 3.3 in the Essential Introduction for details.

For environments where it might be inappropriate to halt program execution when an error is detected, the value -1 or 1 is recommended. If the output of error messages is undesirable, then the value 1 is recommended. Otherwise, if you are not familiar with this parameter, the recommended value is 0. When the value -1 or 1 is used it is essential to test the value of IFAIL on exit.

On exit: IFAIL = 0 unless the routine detects an error or a warning has been flagged (see Section 6).

# 6 Error Indicators and Warnings

If on entry IFAIL = 0 or -1, explanatory error messages are output on the current error message unit (as defined by X04AAF).

Errors or warnings detected by the routine:

IFAIL = 1

On entry, DIST is not valid:  $DIST = \langle value \rangle$ .

IFAIL = 2

On entry, NUM =  $\langle value \rangle$ . Constraint: NUM  $\geq 0$ .

IFAIL = 3

On entry,  $IP = \langle value \rangle$ . Constraint:  $IP \ge 0$ .

#### IFAIL = 4

On entry,  $IQ = \langle value \rangle$ . Constraint:  $IQ \ge 1$ .

IFAIL = 5

On entry, THETA( $\langle value \rangle$ ) =  $\langle value \rangle$ . Constraint: THETA(i)  $\geq$  0.0.

IFAIL = 7

On entry,  $DF = \langle value \rangle$ . Constraint:  $DF \ge 3$ . Input/Output

Communication Array

#### IFAIL = 11

IP or IQ is not the same as when R was set up in a previous call. Previous value of IP =  $\langle value \rangle$  and IP =  $\langle value \rangle$ . Previous value of IQ =  $\langle value \rangle$  and IQ =  $\langle value \rangle$ .

### $\mathrm{IFAIL}=12$

On entry, LR is not large enough,  $LR = \langle value \rangle$ : minimum length required =  $\langle value \rangle$ .

#### IFAIL = 13

On entry, STATE vector has been corrupted or not initialized.

#### $\mathrm{IFAIL}=51$

On entry, sum of THETA(i), for i = 2, 3, ..., IP + IQ + 1 is  $\geq 1.0$ : sum =  $\langle value \rangle$ .

#### IFAIL = -99

An unexpected error has been triggered by this routine. Please contact NAG.

See Section 3.8 in the Essential Introduction for further information.

#### IFAIL = -399

Your licence key may have expired or may not have been installed correctly.

See Section 3.7 in the Essential Introduction for further information.

#### IFAIL = -999

Dynamic memory allocation failed.

See Section 3.6 in the Essential Introduction for further information.

# 7 Accuracy

Not applicable.

# 8 Parallelism and Performance

G05PEF is threaded by NAG for parallel execution in multithreaded implementations of the NAG Library.

Please consult the X06 Chapter Introduction for information on how to control and interrogate the OpenMP environment used within this routine. Please also consult the Users' Note for your implementation for any additional implementation-specific information.

# **9** Further Comments

None.

### 10 Example

This example first calls G05KFF to initialize a base generator then calls G05PEF to generate two realizations, each consisting of ten observations, from an asymmetric GARCH(1, 1) model.

#### 10.1 Program Text

Program g05pefe

```
G05PEF Example Program Text
1
1
     Mark 25 Release. NAG Copyright 2014.
      .. Use Statements ..
1
     Use nag_library, Only: g05kff, g05pef, nag_wp
1
      .. Implicit None Statement ..
     Implicit None
!
      .. Parameters ..
                                       :: lseed = 1, nin = 5, nout = 6
     Integer, Parameter
      .. Local Scalars ..
1
     Real (Kind=nag_wp)
                                        :: gamma
                                        :: df, genid, i, ifail, ip, iq, lr,
     Integer
                                                                                 &
                                           lstate, ltheta, nreal, num, rn, subid
     Logical
                                        :: fcall
     Character (1)
                                        :: dist
!
     .. Local Arrays ..
     Real (Kind=nag_wp), Allocatable :: et(:), ht(:), r(:), theta(:)
                                        :: seed(lseed)
     Integer
     Integer, Allocatable
                                        :: state(:)
1
      .. Executable Statements ..
      Write (nout,*) 'GO5PEF Example Program Results'
     Write (nout,*)
     Skip heading in data file
1
     Read (nin,*)
     Read in the base generator information and seed
!
     Read (nin,*) genid, subid, seed(1)
!
      Initial call to initialiser to get size of STATE array
     lstate = 0
     Allocate (state(lstate))
      ifail = 0
     Call g05kff(genid,subid,seed,lseed,state,lstate,ifail)
1
     Reallocate STATE
     Deallocate (state)
     Allocate (state(lstate))
!
     Initialize the generator to a repeatable sequence
      ifail = 0
     Call g05kff(genid,subid,seed,lseed,state,lstate,ifail)
     Read in sample size and number of realizations
1
     Read (nin,*) num, nreal
     Read in number of coefficients
1
     Read (nin,*) ip, iq
      lr = 2*(ip+iq+2)
      ltheta = iq + ip + 1
      Allocate (theta(ltheta), ht(num), et(num), r(lr))
1
     Read in error distribution
     Read (nin,*) dist
     Read in degrees of freedom if required
1
     If (dist=='T' .Or. dist=='t') Then
  Read (nin,*) df
     End If
     Read in rest of series parameters
1
     Read (nin,*) theta(1:ltheta)
     Read (nin,*) gamma
     Set FCALL for first realization
!
```

## **G05PEF**

```
fcall = .True.
     Generate NREAL realizations
!
     Do rn = 1, nreal
        ifail = 0
        Call g05pef(dist,num,ip,iq,theta,gamma,df,ht,et,fcall,r,lr,state, &
          ifail)
       Display the results
!
       Write (nout,99998) 'Realization Number ', rn
       Write (nout,*) ' I HT(I) ET(I)'
Write (nout,*) ' ------'
       Write (nout,99999)(i,ht(i),et(i),i=1,num)
       Write (nout,*)
       Set FCALL flag for any further realizations
!
       fcall = .False.
     End Do
99999 Format (1X, I5, 1X, F16.4, 1X, F16.4)
99998 Format (1X,A,IO)
   End Program g05pefe
```

### **10.2 Program Data**

G05PEF Example	Program Data
1 1 1762543	:: GENID, SUBID, SEED(1)
10 2	:: NUM,NREAL
1 1	:: IP,IQ
′N′	
0.08 0.2 0.7	:: THETA
-0.4	:: GAMMA

### **10.3 Program Results**

G05PEF Example Program Results

Realization I	Number 1 HT(I)	ET(I)
1 2 3 4 5 6 7 8 9 10	0.6400 0.5336 0.7780 0.6491 0.5670 0.8275 0.6593 0.5639 0.5005 0.4303	0.2790 -0.9098 0.5840 0.6731 -0.9456 -0.0172 -0.2390 0.5980 -0.0032 0.2917
Realization I	Number 2 HT(I)	ET(I)
1 2 3 4 5 6 7 8 9 10	0.3874 0.7594 0.7371 0.6013 1.1133 0.8638 0.8014 0.8013 1.0003 0.9002	$\begin{array}{c} -1.0205 \\ -0.5659 \\ 0.2709 \\ -1.2499 \\ 0.2505 \\ -0.5457 \\ -0.6395 \\ 2.2341 \\ 1.2908 \\ 0.0727 \end{array}$