NAG Library Routine Document

S19AAF

Note: before using this routine, please read the Users' Note for your implementation to check the interpretation of ***bold italicised*** terms and other implementation-dependent details.

1 Purpose

S19AAF returns a value for the Kelvin function $\mathrm{ber}\, x$ via the function name.

2 Specification

```
FUNCTION S19AAF (X, IFAIL)
REAL (KIND=nag_wp) S19AAF

INTEGER            IFAIL
REAL (KIND=nag_wp) X
```

3 Description

S19AAF evaluates an approximation to the Kelvin function $\mathrm{ber}\, x$.

Note: $\mathrm{ber}(-x) = \mathrm{ber}\, x$, so the approximation need only consider $x \geq 0.0$.

The routine is based on several Chebyshev expansions:

For $0 \leq x \leq 5$,

$$\mathrm{ber}\, x = \sum_{r=0} a_r T_r(t), \qquad \text{with } t = 2\left(\frac{x}{5}\right)^4 - 1.$$

For $x > 5$,

$$\mathrm{ber}\, x = \frac{e^{x/\sqrt{2}}}{\sqrt{2\pi x}}\left[\left(1 + \frac{1}{x}a(t)\right)\cos\alpha + \frac{1}{x}b(t)\sin\alpha\right]$$

$$+\frac{e^{-x/\sqrt{2}}}{\sqrt{2\pi x}}\left[\left(1 + \frac{1}{x}c(t)\right)\sin\beta + \frac{1}{x}d(t)\cos\beta\right],$$

where $\alpha = \frac{x}{\sqrt{2}} - \frac{\pi}{8}$, $\beta = \frac{x}{\sqrt{2}} + \frac{\pi}{8}$,

and $a(t)$, $b(t)$, $c(t)$, and $d(t)$ are expansions in the variable $t = \frac{10}{x} - 1$.

When x is sufficiently close to zero, the result is set directly to $\mathrm{ber}\, 0 = 1.0$.

For large x, there is a danger of the result being totally inaccurate, as the error amplification factor grows in an essentially exponential manner; therefore the routine must fail.

4 References

Abramowitz M and Stegun I A (1972) *Handbook of Mathematical Functions* (3rd Edition) Dover Publications

5 Parameters

1: X – REAL (KIND=nag_wp) *Input*

On entry: the argument x of the function.

2: IFAIL – INTEGER *Input/Output*

On entry: IFAIL must be set to 0, -1 or 1. If you are unfamiliar with this parameter you should refer to Section 3.3 in the Essential Introduction for details.

For environments where it might be inappropriate to halt program execution when an error is detected, the value -1 or 1 is recommended. If the output of error messages is undesirable, then the value 1 is recommended. Otherwise, if you are not familiar with this parameter, the recommended value is 0. **When the value -1 or 1 is used it is essential to test the value of IFAIL on exit.**

On exit: IFAIL $= 0$ unless the routine detects an error or a warning has been flagged (see Section 6).

6 Error Indicators and Warnings

If on entry IFAIL $= 0$ or -1, explanatory error messages are output on the current error message unit (as defined by X04AAF).

Errors or warnings detected by the routine:

IFAIL $= 1$

On entry, abs(X) is too large for an accurate result to be returned. On softfailure, the routine returns zero. See also the Users' Note for your implementation.

IFAIL $= -99$

An unexpected error has been triggered by this routine. Please contact NAG.

See Section 3.8 in the Essential Introduction for further information.

IFAIL $= -399$

Your licence key may have expired or may not have been installed correctly.

See Section 3.7 in the Essential Introduction for further information.

IFAIL $= -999$

Dynamic memory allocation failed.

See Section 3.6 in the Essential Introduction for further information.

7 Accuracy

Since the function is oscillatory, the absolute error rather than the relative error is important. Let E be the absolute error in the result and δ be the relative error in the argument. If δ is somewhat larger than the ***machine precision***, then we have:

$$E \simeq \left| \frac{x}{\sqrt{2}} (\mathrm{ber}_1\, x + \mathrm{bei}_1\, x) \right| \delta$$

(provided E is within machine bounds).

For small x the error amplification is insignificant and thus the absolute error is effectively bounded by the ***machine precision***.

For medium and large x, the error behaviour is oscillatory and its amplitude grows like $\sqrt{\frac{x}{2\pi}} e^{x/\sqrt{2}}$.

Therefore it is not possible to calculate the function with any accuracy when $\sqrt{x} e^{x/\sqrt{2}} > \frac{\sqrt{2\pi}}{\delta}$. Note that this value of x is much smaller than the minimum value of x for which the function overflows.

8 Parallelism and Performance

Not applicable.

9 Further Comments

None.

10 Example

This example reads values of the argument x from a file, evaluates the function at each value of x and prints the results.

10.1 Program Text

```
    Program s19aafe

!     S19AAF Example Program Text

!     Mark 25 Release. NAG Copyright 2014.

!     .. Use Statements ..
      Use nag_library, Only: nag_wp, s19aaf
!     .. Implicit None Statement ..
      Implicit None
!     .. Parameters ..
      Integer, Parameter               :: nin = 5, nout = 6
!     .. Local Scalars ..
      Real (Kind=nag_wp)               :: x, y
      Integer                          :: ifail, ioerr
!     .. Executable Statements ..
      Write (nout,*) 'S19AAF Example Program Results'

!     Skip heading in data file
      Read (nin,*)

      Write (nout,*)
      Write (nout,*) '     X           Y'
      Write (nout,*)

data: Do
        Read (nin,*,Iostat=ioerr) x

        If (ioerr<0) Then
          Exit data
        End If

        ifail = -1
        y = s19aaf(x,ifail)

        If (ifail<0) Then
          Exit data
        End If

        Write (nout,99999) x, y
      End Do data

99999 Format (1X,1P,2E12.3)
    End Program s19aafe
```

10.2 Program Data

```
S19AAF Example Program Data
             0.1
             1.0
             2.5
             5.0
             10.0
             15.0
             -1.0
```

10.3 Program Results

```
 S19AAF Example Program Results

      X           Y

    1.000E-01   1.000E+00
    1.000E+00   9.844E-01
    2.500E+00   4.000E-01
    5.000E+00  -6.230E+00
    1.000E+01   1.388E+02
    1.500E+01  -2.967E+03
   -1.000E+00   9.844E-01
```