

NAG Library Routine Document

G01BLF

Note: before using this routine, please read the Users' Note for your implementation to check the interpretation of *bold italicised* terms and other implementation-dependent details.

1 Purpose

G01BLF returns the lower tail, upper tail and point probabilities associated with a hypergeometric distribution.

2 Specification

```
SUBROUTINE G01BLF (N, L, M, K, PLEK, PGTK, PEQK, IFAIL)
  INTEGER          N, L, M, K, IFAIL
  REAL (KIND=nag_wp) PLEK, PGTK, PEQK
```

3 Description

Let X denote a random variable having a hypergeometric distribution with parameters n , l and m ($n \geq l \geq 0$, $n \geq m \geq 0$). Then

$$\text{Prob}\{X = k\} = \frac{\binom{m}{k} \binom{n-m}{l-k}}{\binom{n}{l}},$$

where $\max(0, l - (n - m)) \leq k \leq \min(l, m)$, $0 \leq l \leq n$ and $0 \leq m \leq n$.

The hypergeometric distribution may arise if in a population of size n a number m are marked. From this population a sample of size l is drawn and of these k are observed to be marked.

The mean of the distribution $= \frac{lm}{n}$, and the variance $= \frac{lm(n-l)(n-m)}{n^2(n-1)}$.

G01BLF computes for given n , l , m and k the probabilities:

$$\begin{aligned} \text{PLEK} &= \text{Prob}\{X \leq k\} \\ \text{PGTK} &= \text{Prob}\{X > k\} \\ \text{PEQK} &= \text{Prob}\{X = k\}. \end{aligned}$$

The method is similar to the method for the Poisson distribution described in Knİsel (1986).

4 References

Knİsel L (1986) Computation of the chi-square and Poisson distribution *SIAM J. Sci. Statist. Comput.* **7** 1022–1036

5 Arguments

1: N – INTEGER *Input*
On entry: the parameter n of the hypergeometric distribution.
Constraint: $N \geq 0$.

- 2: L – INTEGER *Input*
On entry: the parameter l of the hypergeometric distribution.
Constraint: $0 \leq L \leq N$.
- 3: M – INTEGER *Input*
On entry: the parameter m of the hypergeometric distribution.
Constraint: $0 \leq M \leq N$.
- 4: K – INTEGER *Input*
On entry: the integer k which defines the required probabilities.
Constraint: $\max(0, L - (N - M)) \leq K \leq \min(L, M)$.
- 5: PLEK – REAL (KIND=nag_wp) *Output*
On exit: the lower tail probability, $\text{Prob}\{X \leq k\}$.
- 6: PGTK – REAL (KIND=nag_wp) *Output*
On exit: the upper tail probability, $\text{Prob}\{X > k\}$.
- 7: PEQK – REAL (KIND=nag_wp) *Output*
On exit: the point probability, $\text{Prob}\{X = k\}$.
- 8: IFAIL – INTEGER *Input/Output*
On entry: IFAIL must be set to 0, -1 or 1. If you are unfamiliar with this argument you should refer to Section 3.4 in How to Use the NAG Library and its Documentation for details.
 For environments where it might be inappropriate to halt program execution when an error is detected, the value -1 or 1 is recommended. If the output of error messages is undesirable, then the value 1 is recommended. Otherwise, if you are not familiar with this argument, the recommended value is 0. **When the value -1 or 1 is used it is essential to test the value of IFAIL on exit.**
On exit: IFAIL = 0 unless the routine detects an error or a warning has been flagged (see Section 6).

6 Error Indicators and Warnings

If on entry IFAIL = 0 or -1, explanatory error messages are output on the current error message unit (as defined by X04AAF).

Errors or warnings detected by the routine:

IFAIL = 1

On entry, $N < 0$.

IFAIL = 2

On entry, $L < 0$,
 or $L > N$.

IFAIL = 3

On entry, $M < 0$,
 or $M > N$.

IFAIL = 4

On entry, $K < 0$,
 or $K > L$,
 or $K > M$,
 or $K < L + M - N$.

IFAIL = 5

On entry, N is too large to be represented exactly as a real number.

IFAIL = 6

On entry, the variance (see Section 3) exceeds 10^6 .

IFAIL = -99

An unexpected error has been triggered by this routine. Please contact NAG.

See Section 3.9 in How to Use the NAG Library and its Documentation for further information.

IFAIL = -399

Your licence key may have expired or may not have been installed correctly.

See Section 3.8 in How to Use the NAG Library and its Documentation for further information.

IFAIL = -999

Dynamic memory allocation failed.

See Section 3.7 in How to Use the NAG Library and its Documentation for further information.

7 Accuracy

Results are correct to a relative accuracy of at least 10^{-6} on machines with a precision of 9 or more decimal digits, and to a relative accuracy of at least 10^{-3} on machines of lower precision (provided that the results do not underflow to zero).

8 Parallelism and Performance

G01BLF is not threaded in any implementation.

9 Further Comments

The time taken by G01BLF depends on the variance (see Section 3) and on k . For given variance, the time is greatest when $k \approx lm/n$ (= the mean), and is then approximately proportional to the square-root of the variance.

10 Example

This example reads values of n , l , m and k from a data file until end-of-file is reached, and prints the corresponding probabilities.

10.1 Program Text

```

Program g01blfe

!      G01BLF Example Program Text

!      Mark 26 Release. NAG Copyright 2016.

!      .. Use Statements ..

```

```

      Use nag_library, Only: g01blf, nag_wp
!      .. Implicit None Statement ..
      Implicit None
!      .. Parameters ..
      Integer, Parameter          :: nin = 5, nout = 6
!      .. Local Scalars ..
      Real (Kind=nag_wp)         :: peqk, pgtk, plek
      Integer                     :: ifail, k, l, m, n
!      .. Executable Statements ..
      Write (nout,*) 'G01BLF Example Program Results'
      Write (nout,*)

!      Skip heading in data file
      Read (nin,*)

!      Display titles
      Write (nout,*) '   N   L   M   K   PLEK   PGTK   PEQK'
      Write (nout,*)

!      Loop over all dta
d_lp: Do
      Read (nin,*,Iostat=ifail) n, l, m, k
      If (ifail/=0) Then
!          All data processed
          Exit d_lp
      End If

!          Calculate probability
      ifail = 0
      Call g01blf(n,l,m,k,plek,pgtk,peqk,ifail)

!          Display results
      Write (nout,99999) n, l, m, k, plek, pgtk, peqk
End Do d_lp

99999 Format (1X,4I4,3F10.5)
      End Program g01blfe

```

10.2 Program Data

```

G01BLF Example Program Data
  10  2  5  1   : N, L, M, K
  40 10  3  2
 155 35 122 22
1000 444 500 220

```

10.3 Program Results

G01BLF Example Program Results

| N | L | M | K | PLEK | PGTK | PEQK |
|------|-----|-----|-----|---------|---------|---------|
| 10 | 2 | 5 | 1 | 0.77778 | 0.22222 | 0.55556 |
| 40 | 10 | 3 | 2 | 0.98785 | 0.01215 | 0.13664 |
| 155 | 35 | 122 | 22 | 0.01101 | 0.98899 | 0.00779 |
| 1000 | 444 | 500 | 220 | 0.42429 | 0.57571 | 0.04913 |
